Dimensions Effect of Sudden Contraction Vibrated Pipe Conveying Fluid With and Without Heating

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Abstract:

Changing geometrical dimensions (diameters&lengths) of sudden contraction of vibrated pipe conveying fluid with and without heat flux was studied in this paper. The flowing fluid in the pipe is laminar. The governing equations of motion for this system are derived by using beam theory. The effect of external force that applied at contraction (mid length) of the pipe is studied using the transfer matrix method. By using this technique the natural frequencies, mode shapes, deflection, slope, bending moment, shear force, velocity, and pressure can be computed for different cases of pipe conveying fluid with and without heat flux, also the effect of forced vibration on these parameters are presented. Different types of supports are used to show the effect of changing the support's type on the behavior of this system at different fluid velocities, heat flux and geometrical dimensions of this pipe. Also the effect of change the values of fluid velocities and heat flux on the Coriolis and compressive force at different geometrical dimensions of this pipe are studied. MATLAB program are used in this study. The results of this study are compared with the results of the previous investigation. The comparisons show good agreement.

Keywords: pipe conveying fluid, sudden contraction pipe, vibration in pipe, geometrical dimension effect of pipe, laminar flow, pipe under heat flux.

Introduction:

Heat flux constitutes an important limitation on the operation of boiling heat transfer systems. In heat flux controlled systems such as nuclear reactors, the consequence is a substantial increase in wall temperature, which may result in a physical failure of heat transfer systems. Also changing geometrical dimensions (diameters&lengths) of pipe have many effects on the system such as changing the natural frequencies and all other pipe's behavior which leads to change the safety life of the system.

Many components as (steam generators, condensers, piping systems, and nuclear fuels) are subjected to high axial or cross flow which could often cause vibration problems, resulting in wear and fretting damage to those systems. Al-Jumaily and Al-Ammri (1986) studied the vibrations of pipes conveying flowing fluids with compliant boundaries and they found the first three natural frequencies. The rotating pipe conveying fluid considered by Bogdeviclus (2003). He derived the equation of motion of the pipe by using the finite element method. An experimental study by Sinha et al. (2005) for flow-induced excitation in a pipe conveying fluid, they measured the structural responses using vibration transducers all along the pipe length and then estimate the flow induced excitation forces. A theoretical investigation of the free vibration characteristics of a gas pipeline model was studied by El-Kafrawy et al. (2007), the solution is based on the finite element method by the PC software "CAEPIPE", the natural frequencies and 20 mode shapes of a supported-end buried gas pipeline model are calculated. Alwan (2007) studied the effect of heat flux on induced vibrations of a fluid flow in a pipe with a restriction, he found the effect of restriction in pipe on the system behavior also found the natural frequencies and mode shapes for this system. Ryu et al. (2008) studied the vibration and dynamic stability of cantilevered pipes conveying fluid on elastic foundations, the critical flow velocity and stability maps of the pipe are obtained for various elastic foundation para-meters, mass ratios of the pipe, and structural damping coefficients. Salim (2008) studied straight pipe conveying fluid combined with vibration, he found the effect of support type (flexible, simply and rigid) and some design parameters like pipe material, wall thickness, and fluid velocity on the natural frequencies and corresponding mode shapes. Kim et al. (2009) investigated methods of analyzing fluid-induced vibration in consideration of the cooling effect, they found that the natural frequency of the system tends to change because of the changes of the properties of materials even when the flux is constant inside pipe, and the vibration behavior of the system was compared to that in case of normal temperature to analyze how much influence cooling effect has on the vibration behavior of the system. Al-Hashimy (2009) studied the effect of end conditions (flexible, simply and rigid) and pipe material on the vibration characteristics of a pipe conveying fluid with different cross sections and without heating, he found the natural frequencies and the corresponding mode shapes. In this research effect of changing geometrical dimensions (diameters&lengths) of sudden contraction vibrated pipe conveying fluid with and without heating are studied to analyze how much influence geometrical dimensions effects on vibration behavior of system.

Governing Equations Of Motion:

The dimensionless equation of motion for forced vibration of undamped horizontal pipe conveying uniform axial flow, based on the beam theory may be written as:

$$\overset{iv}{\bar{Y}}+\left(\gamma+\bar{U}_1.\bar{U}_2\right)\bar{Y}''+\bar{N}.L_m\bar{Y}''+2.\bar{U}.\beta.\dot{\bar{Y}}+\left(1/L_m\right)\ddot{\bar{Y}}=F\left(\bar{X},\tau\right) \quad (1)$$

where: $N=A_p \cdot E_p \cdot \alpha \cdot \Delta T$, $\bar{X}=x/L_m$, $\bar{Y}=y/L_m$, $\bar{U}=(m_f/E.I)^{1/2}\cdot u_f \cdot L_m$, $\tau=\left(EI/(m_f+m_p)\right)^{1/2}\left(t/L_m^2\right)$

The second and third terms from the left hand side of equation of motion represent the compressive force plus the Corioles force, equal to:

$$W\left(x,t\right)=\left(m_f \cdot u_1 \cdot u_2+P \cdot A_p+N\right)(\partial^2 y/\partial x^2)+\left(2 \cdot m_f \cdot u_1\right).(\partial^2 y/\partial x \partial t) \quad (2)$$

Investigation of The Flow Stream:

The value of inlet velocity (u_1) can be found from inlet Reynolds number where:

$$u_1=\mu.Re/\left(\rho.D1\right) \quad (3)$$

The velocity (u_2) through the contraction of the pipe can be determined by using the formula:

$$u_2=\alpha\, u_1 \quad (4)$$

Since the flow get out to atmosphere; therefore, the out let pressure (P_2) = 1atm, and the inlet pressure to the pipe (P_1) can be found from the energy equation as follows:

$$P_1=P_2+\left((u_2^2-u_1^2)/2+\text{losses}\right)*\rho+(Z_2-Z_1)*(\rho.g) \quad (5)$$

where: $z_1=z_2=0$ (For horizontal pipe) & losses=$P_{L1}+P_{L2}+P_{Lc}$

The Losses At Contraction Section Of Pipe Conveying Fluid (P_{lc}):

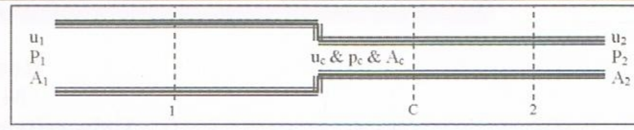

Figure (1) represents sudden contraction pipe conveying fluid

This figure shows the flow in a sudden contraction, mass balance and momentum balance between the cross sections (1, C and 2) give:

Mass balance:

$$u_1.A_1=u_2.A_2=u_C.A_C \therefore u_C=\frac{A_2}{A_C}.u_2 \quad (6)$$

Momentum balance between (C and 2):

$$Pc.Ac+Pc\left(A_2-Ac\right)-P_2.A_2=\rho.u_2.A_2.\left(u_2-u_c\right) \quad (7)$$

Where the term $\left[Pc\left(A_2-Ac\right)\right]$ comes from the effect of fluid on the wall at the contraction section.

$$P_2-P_c=\rho.u_2^2\left(\frac{A_2}{A_c}-1\right) \quad (8)$$

Applying the Bernoulli's equation between two points "C" and "2", gives:

$$\frac{1}{2}.\rho.u_c^2+P_c=\frac{1}{2}.\rho.u_2^2+P_2+\Delta P \quad (9)$$

Substitute equation (8) in equation (9) gives:

$$\frac{1}{2}.\rho.u_2^2[(\frac{A_2}{A_c})-1]=\frac{1}{2}.\rho.u_2^2[(\frac{A_2}{A_c})^2-1]-\Delta P \quad (10)$$

$$\therefore \Delta P=\frac{1}{2}.\rho.u_2^2.[(\frac{A_2}{A_c})-1]^2$$

Let: $C_c=\left[(\frac{A_2}{A_c})-1\right]^2$

$$\therefore \Delta P=\frac{1}{2}.\rho.u_2^2.C_c=P_{Lc} \quad (11)$$

The Transfer Matrix Method:

In this method, system of equation can be converted to a mathematical model consist of number of stations represented by point matrix where the mass concentrated at each station, each station joint with massless element which represented by field matrix, then we can found the equations of {deflection(Y), slope(θ), bending moment(M), shear force(V), velocity(U), pressure(P)} for the vibrated pipe conveying fluid. The analytical solution used to solve these equations. Where these equations are:

The equations for field matrix:

$$\bar{Y}_i^L = \bar{Y}_{i-1}^R - \frac{\bar{q}_{i-1}^R}{L_n}\left[L_i + \frac{\bar{N}_{i-1}^R}{\varphi}\left(\frac{L_i^3}{6.EI} - \frac{x.L_i}{GA_n}\right)\right] - \bar{M}_{i-1}^R\left(\frac{L_i^2}{2.\phi L_n .EI}\right) - \bar{V}_{i-1}^R \frac{1}{\varphi . L_n}\left(\frac{L_i^3}{6.EI} - \frac{x.L_i}{GA_n}\right) + \frac{\bar{W}_i}{\varphi . L_n}\left(\frac{L_i^3}{48.EI} - \frac{x.L_i}{GA_n}\right) \quad (12)$$

$$\bar{\theta}_i^L = \bar{\theta}_{i-1}^R\left(1 + \bar{N}_{i-1}^R \cdot \frac{L_i^2}{2\cdot\phi\cdot EI}\right) + \bar{M}_{i-1}^R \cdot \frac{L_i}{\varphi\cdot EI} + \bar{V}_{i-1}^R \cdot \frac{L_i^2}{2\cdot\phi\cdot EI} - \bar{W}_i \cdot \frac{L_i^2}{8\cdot\phi\cdot EI} \quad (13)$$

$$\bar{M}_i^L = \bar{\theta}_{i-1}^R \cdot \bar{N}_{i-1}^R .(L_i .\phi/\varphi) + \bar{M}_{i-1}^R + \bar{V}_{i-1}^R \cdot (L_i .\phi/\varphi) - (\bar{W}_i/\varphi)\cdot(L_i .\phi/2) \quad (14)$$

$$\bar{V}_i^L = \bar{\theta}_{i-1}^R\left[\bar{N}_{i-1}^R\left(1 - \bar{N}_i^L \frac{L_i^2}{2\varphi\times EI}\right) - \bar{N}_i^L\right] - \bar{M}_{i-1}^R\left(\frac{L_i}{\phi.EI}\right) + \bar{V}_{i-1}^R\left(1 - \bar{N}_i^L \frac{L_i^2}{2\varphi\times EI}\right) - \bar{W}_i\left(1 - \bar{N}_i^L \frac{L_i^2}{8.\varphi.EI}\right) \quad (15)$$

$$\bar{U}_i = [m_f/EI]^{1/2}.U_i.L_m \qquad \bar{P}_i = P_i/P_{inlet} \quad (16)$$

The equations for the supported node:

$$\bar{Y}_i^L = \bar{Y}_i^R \,\&\, \bar{\theta}_i^L = \bar{\theta}_i^R \,\&\, \bar{M}_i^L = \bar{M}_i^R \quad (17)$$

$$\bar{V}_i^R = -\left[(m_p + m_f)_i \cdot \Omega^2 - K\right]\cdot(L_m^3/(EI)_m)\cdot \bar{Y}_i + \bar{V}_i^L \quad (18)$$

$$\bar{U}_i^L = \bar{U}_i^R \,\&\, \bar{P}_i^L = \bar{P}_i^R \quad (19)$$

The equations for the sudden contraction node:

$$\bar{Y}_i^L = \bar{Y}_i^R \,\&\, \bar{\theta}_i^L = \bar{\theta}_i^R \,\&\, \bar{M}_i^L = \bar{M}_i^R \quad \&$$

$$\bar{Y}_i^L = \bar{Y}_i^R \,\&\, \bar{\theta}_i^L = \bar{\theta}_i^R \,\&\, \bar{M}_i^L = \bar{M}_i^R \quad \&$$
$$\bar{U}_i^L = \eta.\bar{U}_i^R \quad \&$$
$$\bar{P}_i^L = \bar{P}_i^R + (C_c.\rho.u^2/(2.P_{inlet})) \quad (20)$$

The equations for the particular node:

$$\bar{Y}_i^L = \bar{Y}_i^R \,\&\, \bar{\theta}_i^L = \bar{\theta}_i^R \,\&\, \bar{M}_i^L = \bar{M}_i^R \quad (21)$$

$$\bar{V}_i^R = -(m_p + m_f)_i \cdot \Omega^2 \cdot (L_m^3/(EI)_m)\cdot \bar{Y}_i - F_o \cdot (L_m^2/(EI)_m) + \bar{V}_i^L \quad (22)$$

$$\bar{U}_i^L = \bar{U}_i^R \,\&\, \bar{P}_i^L = \bar{P}_i^R \quad (23)$$

Results And Discussion:

1-Changing Diameters:

*The values of the natural frequencies for pipe conveying fluid with various velocities and heat flux and for different kinds of supports (simply, flexible, rigid) and for different diameters can be found from drawing relation between the deflection of the pipe and the excitation frequency where each peak of this drawing represent the natural frequency. The values of the natural frequencies for different cases of pipe diameters, supports, heat flux and fluid velocity are given in appendix (B) tables (I-1) to (I-3). It can be notice from these tables the effects of changing pipe diameters on the values of natural frequencies as follows:

**For the case of the vibrated pipe without fluid and heat flux, the values of natural frequencies decrease with the decreasing of the large diameter of the pipe (first part) at constant small diameter of the pipe (second part) because the pipe stiffness decreases, where the stiffness has direct proportion with the second moment of area (I) where $(K = 48\,EI/L^3)$ and since $(I = (\pi/64) * (D_{out}^4 - D_{in}^4))$; Therefore, the stiffness (K) has direct proportion with the pipe diameter $(K \alpha D)$, hence decreasing the pipe diameter leads to decreasing the pipe stiffness and natural frequencies. At the same case (without fluid and heat flux) but at decreasing the small diameter of the pipe (second part) with constant large diameter (first part) the values of natural frequencies decrease more than the previous because the pipe stiffness (K) decreases more than the previous.

**For the case of vibrated pipe conveying fluid without heat flux, the natural frequencies decrease with decreasing the large diameter at constant small diameter because the pipe stiffness decreases (as explained before), this happened for all Reynolds number and supports pipe.

**For the case of vibrated pipe conveying fluid and exposed to heat flux, the natural frequencies decrease with decreasing the large diameter at constant small diameter because the pipe stiffness decreases (as explained before), this happened for all supports except flexible support where these values increase with the decreasing of the large diameter at constant small diameter of the pipe.

**At decreasing (0.25inch) from the diameter of the first and the second parts of the pipe it can be seen decreasing the values of the natural frequencies more than that at decreasing the pipe diameters to half original dimensions (1 inch for the first part of the pipe and 0.5 inch for the second part). Also it can be noticed that the smallest natural frequencies can be obtained at decreasing small diameter with constant large diameter (i.e. at the pipe diameters equal to (1)inch for the first part and (0.25)inch for the second part of the pipe) because at this case the pipe stiffness for the second part of the system (small diameter) will be very small whereas at the first part of the system (large diameter) the pipe stiffness (K) will be very large comparing with the other part of the system, hence that leads to weakness in the system hence broken the system at the values of the natural frequencies less than the other cases.

**When comparing the case of the pipe conveying fluid without heat flux with the case of the pipe conveying fluid and exposed to heat flux , it can be noticed that the values of the natural frequencies decrease when applying heat flux for all supports except flexible support where some values of the natural frequencies increase when applying heat flux, but at reducing the pipe diameters to half dimensions (0.5-0.25)inch it can be seen disappearing this phenomenon from the flexible support (where all values of the natural frequencies will decrease at applying heat flux). Also from the comparison between these two cases it can be noticed that the decreasing in the values of the natural frequencies (which result from the effect of heat flux) are small comparing with the decreasing before reducing the diameters, to explain this show the following comparison:

For pipe with diameters (1-0.5)inch (simply support)&heat flux=0 gives: $\omega_2 = 772$

For pipe with diameters (1-0.5)inch (simply support)&heat flux=10kW/m^2 gives: $\omega_2 = 764.5$

For pipe with diameters (0.5-0.25)inch (simply support)&heat flux=0 gives: $\omega_2 = 394$

For pipe with diameters (0.5-0.25)inch (simply support)&heat flux=10kW/m^{2}gives: $\omega_2 = 393$

$\therefore$ The reduction in the natural frequency for pipe with diameters (1-0.5)inch =7.5

The reduction in the natural frequency for pipe with diameters (0.5-0.25)inch =1

The cause of this disproportion accrue to that the fluid velocity which flows in pipe dependent on the values of Reynolds number (Re) and pipe diameter at constant other fluid properties $(u = Re^* \mu/(\rho^* D))$, and since Reynolds number is constant before and after reducing diameters; Therefore, fluid velocity will emulate with diameter only. Also it can be seen from the above low that velocity have inverse proportion with pipe diameter; Therefore, reducing the diameter leads to increasing fluid velocity and this leads to increasing the pipe cooling and absorb more quantity of heat, hence reducing the effect of heat flux on the pipe and on the values of the natural frequencies (where the natural frequency increases with decreasing heat flux because pipe stiffness increases), hence decreasing the values of the natural frequencies at applying heat flux on the pipe with diameters (0.5-0.25)inch are less than decreasing the values of the natural frequencies at applying heat flux on the pipe with diameters (1-0.5)inch.

**At the case of the pipe conveying fluid and exposed to heat flux, it can be seen that all values of the natural frequencies decrease with the increasing of heat flux for all supports except flexible support where some of values of second and third natural frequencies increase with increasing heat flux but reducing the pipe diameters to half dimensions (0.5-0.25)inch leads to disappearing this phenomenon from the flexible support (where all values of the natural frequencies will decrease with the increasing of heat flux).

**At the case of heated pipe conveying fluid it can be noticed that increasing Reynolds number leads to increasing natural frequencies for all supports except second and third values of natural frequencies for flexible support at pipe diameters equal to (1-0.5)inch where they decrease with increasing Reynolds number. But, it's found that altering pipe diameters may affect pipe stiffness and in turn affect values of the natural frequencies. Tables (I-3) presents first, second, and third natural frequencies for Reynolds number of (250 to 1500) for different pipe diameters and for heat flux=(10-15-20)kW/m^2, it can be seen disappearing this phenomenon with reducing pipe diameters where all natural frequencies increase with increasing Reynolds number (there are no decreasing in natural frequencies with increasing Reynolds number) at pipe diameters equal to (0.5-0.25)inch.

*Figs. (2) and table (I-2) show that the values of natural frequencies(1st,2nd,3rd) for the case of vibrated pipe conveying fluid without heat (for all supports) are still constant with the increasing of Reynolds number for all values of pipe diameters because increasing Reynolds number leads to increasing fluid velocity and this increase (when there is no heat) doesn't affect the properties of pipe material (stiffness, mass). To prove this phenomenon, a comparison is made for the results of the natural frequencies between the present work (for different values of pipe diameters) and the last investigation of Alwan (2007) which used Fortran program and takes straight pipe with restriction have diameters equal (1) inch. It can be seen that the same phenomenon exists in both works.

*Figs.(3) to (8) represent first, second, and third mode shapes for different pipe diameters and different kinds of supports and for the case of vibrated pipe without fluid and heat and the case of vibrated pipe conveying fluid without heat flux and the case of heated pipe conveying fluid. These cases were at range of Reynolds number (250) to (1500) and range of heat flux (10-15-20) kw/m^2. For all cases it can be seen that the maximum deflection occurs at the pipe diameters equal to (1-0.25) inch that's because the first part of the pipe is very stiff compared with the second part of the pipe that's make the stresses centralize at the second part of the pipe more than that the other cases of the pipe diameters that leads to the deflection in this case is more than the other cases. Sometimes, the maximum deflection occurs at the pipe diameter equal to (0.5-0.25) inch that's because the stiffness of pipe for this case is the smallest compared with the other pipe diameters as follows:

The stiffness for the pipe diameters (1-0.5)inch =16554.22272 N/m

The stiffness for the pipe diameters (0.75-0.5)inch = 12568.49275 N/m

The stiffness for the pipe diameters (1-0.25)inch = 1956.81595 N/m

The stiffness for the pipe diameters (0.75-0.25)inch = 1805.15207 N/m

The stiffness for the pipe diameters (0.5-0.25)inch = 1542.02363 N/m

Where if the effect of stresses more than the effect of stiffness the maximum deflection occurs at the pipe diameters equal to (1-0.25) inch else the maximum deflection occurs at the pipe diameters equal to (0.5-0.25)inch.

* Figs.(9) to (11) show bending moment, slope, and shear force for different supported pipe at its natural frequencies with various heat flux and fluid velocity, it can be noticed from these figures that the (bending moment, slope, shear force) increase either at the pipe diameters equal to (1-0.25)inch or at the pipe diameters equal to (0.25-0.5)inch, the cause of this is the same cause as explained for the maximum deflection.

* Fig. (12) shows Coriolis and compressive force for different supported pipe conveying fluid due to forced vibration at mid length with various velocities and heat flux. It may be noticed that decreasing the second pipe diameter (small diameter) has more effect on Coriolis and compressive force than decreasing the first part of the pipe (large diameter). Also the largest Coriolis and compressive force occurs when decreasing the pipe diameters to half that's because decreasing the pipe diameters leads to increasing the Reynolds number and that leads to increasing fluid velocity $(Re = \rho * u * D/\mu)$ and since the fluid velocity has direct proportion with the Coriolis and compressive force (see Coriolis equation); Therefore, decreasing the pipe diameter leads to increasing Coriolis and compressive force. Also when comparing fig.(12 b&c&d) which represent different pipe supports, it can be seen that the kind of support don't affect Coriolis and compressive force because the effect of support don't enter in the Coriolis equation. Fig. (12 a) represent the effect of heat flux on the Coriolis and compressive force, where applying heat flux leads to increase Coriolis and compressive force for all pipe diameters because the heat flux leads to increase the thermal force (N) and since the thermal force has direct proportion with the Coriolis and compressive force (see its equation), hence increasing thermal force leads to increasing Coriolis and compressive force.

2-Changing Length:

* The values of the natural frequencies for pipe conveying fluid with various velocities and heat flux and for different kinds of supports (simply, flexible, rigid) and for different lengths can be found from drawing relation between the deflection of the pipe and the excitation frequency where each peak of this drawing represent the natural frequency. The values of the natural frequencies for different cases of pipe diameters, supports, heat flux and fluid velocity are given in appendix (B) tables (II-1) to (II-3). It can be notice from these tables the effects of changing pipe lengths on the values of natural frequencies as follows:

**For the case of the vibrated pipe without fluid and heat flux, the values of natural frequencies decrease with the increasing of the pipe length because the pipe stiffness decreases, the stiffness (K) has inverse proportion with the pipe length $(K \alpha 1/L)$ where $(K = 48 EI/L^3)$, hence increasing the pipe length leads to decreasing the pipe stiffness and natural frequencies.

**For the case of vibrated pipe conveying fluid without heat flux and the case of vibrated pipe conveying fluid and exposed to heat flux, the natural frequencies decrease with increasing pipe length because the pipe stiffness decreases (as explained before).

**when comparing the case of the pipe conveying fluid without heat flux with the case of the pipe conveying fluid with heat flux it can be seen decreasing the values of the natural frequencies at applying heat flux on the pipe with the length (1.3m) are greater than the decreasing in the values of natural frequencies at applying heat flux on the pipe with the length (2m), to explain this show the following comparison:

For pipe with lengths (2)m (simply support)&heat flux=0 gives: $\omega_2 = 192.5$ rad/sec

For pipe with lengths (2)m (simply support)&heat flux=10kW/m^2 gives: $\omega_2 = 190.5$ rad/sec

For pipe with lengths (1.3)m (simply support)&heat flux=0 gives: $\omega_2 = 455$ rad/sec

For pipe with lengths (1.3)m (simply support)&heat flux=10kW/m^{2}gives: $\omega_2 = 450.5$ rad/sec

$\therefore$ The reduction in the natural frequency for pipe with lengths (2)m =2

The reduction in the natural frequency for pipe with lengths (1.3)m =4.5

The cause of this disproportion accrue to that the pipe with length (2m) has fluid quantity more than the other pipe lengths; therefore, it observe large quantity of heat, also the fluid flows with large distance in the pipe with length (2m) than the pipe with other lengths hence the period that the fluid will be in touch with the inner pipe surface in the length of (2m) more than the other lengths, that's leads to increase the heat transfer from the pipe surface to the fluid hence the fluid observe large quantity of heat; therefore, the cooling in the pipe with length (2m) will be more than the cooling in the other length (1, 1.3, 1.5)m hence decreasing the heat effect on the pipe with length (2m) and since natural frequency decreases with the increasing the heat flux (as explained before); therefore, decreasing the values of the natural frequencies when applying heat at the pipe conveying fluid with length (2m) will be less than the decreasing in the values of the natural frequencies at applying heat flux on the pipe conveying fluid with other lengths.

**At the case of the pipe conveying fluid and exposed to heat flux, it can be seen that all values of the natural frequencies decrease with the increasing of heat flux for all supports except flexible support where some of values of second and third natural frequencies increase with increasing heat flux but increasing the pipe lengths to twice (2m) leads to disappearing this phenomenon from the flexible support (where all values of natural frequencies will decrease with the increasing of heat flux).

**At the case of heated pipe conveying fluid it can be noticed that increasing Reynolds number leads to increasing the natural frequencies for all supports except the second and third values of natural frequencies for flexible support for some pipe lengths where they decrease with increasing Reynolds number. But, it's found that increasing pipe lengths may affect the pipe stiffness and in turn affect the values of the natural frequencies. The tables (II-3) presents first, second, and third natural frequencies for Reynolds number of (250 to 1500) for different pipe lengths and for heat flux=(10-15-20)kW/m^2, it can be seen disappearing this phenomenon with increasing pipe lengths where all natural frequencies increase with increasing Reynolds number (there are no decreasing in natural frequencies with increasing Reynolds number) at pipe lengths equal to (2m).

**For all cases, it can be noticed the convergence with the values of natural frequencies for simply support and flexible support also more decreasing with the values of natural frequencies for the length (1.5m) and (2m) comparing with the length (1.3m), that's because the increasing in the pipe length leads to increase the mass system and since the mass has inverse proportion with the natural frequencies $\left(\omega_n = \sqrt{\text{stiffness/ mass}}\right)$; therefore, increasing the pipe length leads to decreasing the values of natural frequencies. Also increasing the pipe length leads to large divergence for the supported point from the midpoint of the pipe; therefore, the pipe become flexible and increase the ability of vibration because it hasn't a support point discontinue its motion, that's mean increasing the pipe length leads to decrease its effects with the supports at each ends; therefore, it can be seen convergence the values of natural frequencies for simply and flexible supports and in the same time large reducing the values of the natural frequencies at increasing the pipe length. While for rigid supports it can be seen reducing the values of natural frequencies at increasing the length because increasing the mass system (as explained before).

**For the case of heated pipe conveying fluid with different lengths, it can be noticed that at the same heat flux and with increasing Reynolds number (Re), the values of the natural frequencies for the pipe length (2m) increase more than that the values of natural frequencies for the other pipe lengths (1, 1.3, 1.5)m, the cause are that the pipe with length (2m) has fluid quantity more than the other pipe lengths; therefore, it observe large quantity of heat, also the fluid flows with large distance in the pipe with length (2m) than the pipe with other lengths (1, 1.3, 1.5)m hence the period that the pipe will be in touch with the pipe surface in the length of (2m) more than the other lengths, that's leads to observed the most heat of the pipe and hence cooling the pipe; therefore, increasing Reynolds number (Re) (increasing the fluid velocity) has little effect on cooling the pipe because the most cooling occurs as a results of passing large quantity of fluid and as a results of touching between the fluid and pipe for large distance; therefore, the values of the natural frequencies for pipe lengths (2m) have little increase comparing with values of the natural frequencies for other pipe lengths (1, 1.3, 1.5)m.

*Figs.(13) and table (II-2) show that the values of natural frequencies (1st,2nd,3rd) for the case of vibrated pipe conveying fluid without heat are still constant with the increasing of Reynolds number for all values of pipe lengths because increasing Reynolds number leads to increasing fluid velocity and this increase (when there is no heat) doesn't affect the properties of pipe material (stiffness, mass). To prove this phenomenon, a comparison is made for the results of the natural frequencies between the present work (for different values of pipe lengths) and the last investigation of Alwan (2007) which used Fortran program and takes straight pipe with restriction have length equal to (1m). It can be seen that the same phenomenon exists in both works.

*Figs.(14) to (19) represent first, second, and third mode shapes for different pipe lengths and different kinds of supports and for the case of vibrated pipe without fluid and heat and the case of vibrated pipe conveying fluid without heat flux and the case of heated pipe conveying fluid. These cases were at range of Reynolds number (250) to (1500) and range of heat flux (10-15-20) kw/m^2. For all cases it can be seen that the maximum deflection occurs at the pipe lengths equal to (2m) because increasing the pipe length leads to decreasing the pipe stiffness $\left(K = 48\,EI/L^3\right)$ and that's lead to increasing the deflection.

* Figs.(20) to (22) show bending moment, slope, and shear force for different supported pipe at its natural frequencies with various heat flux and fluid velocity,

* Fig.(23) shows Coriolis and compressive force for different supported pipe conveying fluid due to forced vibration at mid length with various velocities and heat flux. It may be noticed that increasing Coriolis and compressive force occurs when increasing the pipe length that's because increasing the pipe length leads to increasing the fluid mass in the pipe and since the fluid mass has direct proportion with the Coriolis and compressive force (see Coriolis equation); Therefore, increasing the pipe length leads to increasing Coriolis and compressive force. Also when comparing fig.(23 b&c&d) which represent different pipe supports, it can be seen that the kind of support don't affect Coriolis and compressive force because the effect of support don't enter in the Coriolis equation. Fig.(23 a) represent the effect of heat flux on the Coriolis and compressive force, where applying heat flux leads to increase Coriolis and compressive force for all pipe lengths because the heat flux leads to increase the thermal force (N) and since the thermal force has direct proportion with the Coriolis and compressive force (see its equation), hence increasing thermal force leads to increasing Coriolis and compressive force.

Conclusions:

This investigation shows that changing geometrical dimensions (diameters&lengths) of sudden contraction pipe have many effects on the system behavior where it's changing the (natural frequencies, deflection, slope, bending moment, shear force, fluid velocity, Coriolis and compressive force) as follows:

For Changing Diameters:

1- Increasing the values of natural frequencies with increasing the pipe diameters.

2- Increasing the effect of heat flux on the vibrated system with decreasing the pipe diameters.

3- Least natural frequency can be obtained when the ratio between first and second part of pipe equal to (1:4).

4- For the case of the vibrated pipe without fluid and heat flux all natural frequencies decreases for all supports (simply, flexible, rigid) when pipe diameters equal to (0.5-0.25) inch, also when increasing heat flux all natural frequencies decreases for all supports when pipe diameters equal to (0.5-0.25) inch. For the case of heated pipe conveying fluid when increasing Reynolds number all natural frequencies increases at pipe diameters equal to (0.5-0.25) inch.

5- The maximum (deflection, slope, bending moment, shear force) occurs either at pipe diameters equal to (1-0.25) inch or at (0.5-0.25) inch.

6- Increasing Coriolis and compressive force with decreasing pipe diameters; also reducing the second part of pipe diameter has more effect on Coriolis and compressive force than reducing the first part of pipe diameter.

For Changing Lengths:

1- Decreasing the values of natural frequencies with increasing the pipe lengths.

2- The natural frequencies still constant with increasing Reynolds number for the case of vibrated pipe conveying fluid without heat flux at all pipe lengths.

3- For the case of vibrated pipe conveying fluid and exposed to heat flux all natural frequencies increasing with the increasing of Reynolds number for all supports at the pipe length equal to (2m).

4- Increasing pipe length leads to decreasing values of natural frequencies and reducing its effect with supports kind.

5- Decreasing the effect of heat flux on the vibrated system with increasing the pipe lengths.

6- At increasing the lengths of the heated pipe conveying fluid, the effect of increasing Reynolds number at the same heat flux leads to reduce the rate of increasing the values of the natural frequencies.

7- The maximum deflection occurs at the pipe length equal to (2m).

8- Increasing Coriolis and compressive force with increasing pipe lengths, also Coriolis and compressive force don't affect on supports kind, also applying heat flux leads to increasing the Coriolis and compressive force.

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تأثر الابعاد في التقلص المفاجئ في الانابيب المهتزة المحتوية على سوائل مع او بدون تسخين

رؤى ياسين حمودي

مدرس مساعد

قسم الهندسة الميكانيكية / الجامعة التكنولوجية

الخلاصة:

تم دراسة تأثير تغيير الابعاد الهندسية (الاقطار والاطوال) لانبوب ذو تقلص مفاجيء ومعرض للاهتزاز ويجري فيه مائع ومعرض الى فيض حراري. جريان المائع في الانبوب هو طباقي. معادلة الحركة لهذا النظام اشتقت من نظرية العتبات (beam). تم دراسة تاثير القوة الخارجية المسلطة عند التقلص (في منتصف الطول) للانبوب باستخدام طريقة المصفوفة الانتقالية. من خلال هذه التقنية يمكن حساب الترددات الطبيعية, نسق الاهتزاز, الانحراف, الميل, عزم الانحناء, قوة القص, السرعة والضغط لمختلف حالات الانبوب الذي يجري فيه مائع (بوجود وعدم وجود فيض حراري), كذلك تم عرض تاثير الاهتزاز القسري على هذه الثوابت. انواع مختلفة من التثبيتات استخدمت لتبين تاثير تغيير نوع التثبيت على سلوك النظام عند قيم مختلفة من سرع المائع والفيض الحراري والابعاد الهندسية. كذلك درس تاثير تغيير قيم سرع المائع والفيض الحراري على قوى (Coriolis and compressive) عند مختلف الابعاد الهندسية. تم استخدام برنامج MATLAB في هذه الدراسة. تمت مقارنة النتائج التي تم الحصول عليها من هذه الدراسة مع النتائج التي تم الحصول عليها من بحث سابق. هذه المقارنات بينت تطابق جيد.

APPENDIX(A)

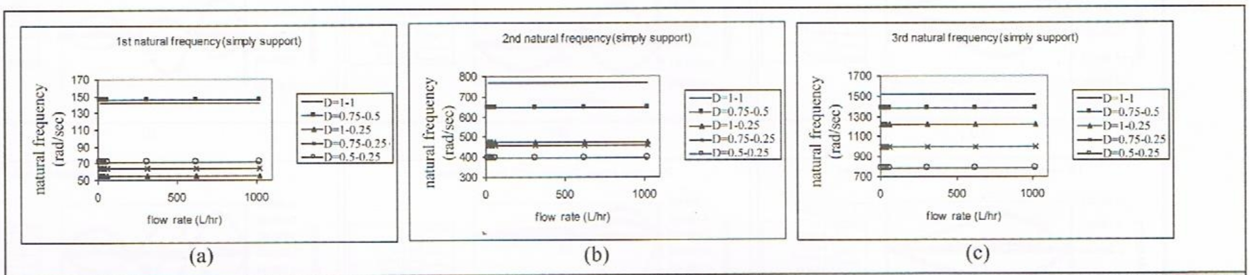


Fig.(2) Natural frequencies for pipe conveying fluid without heat flux and with various flow rates and various pipe diameters

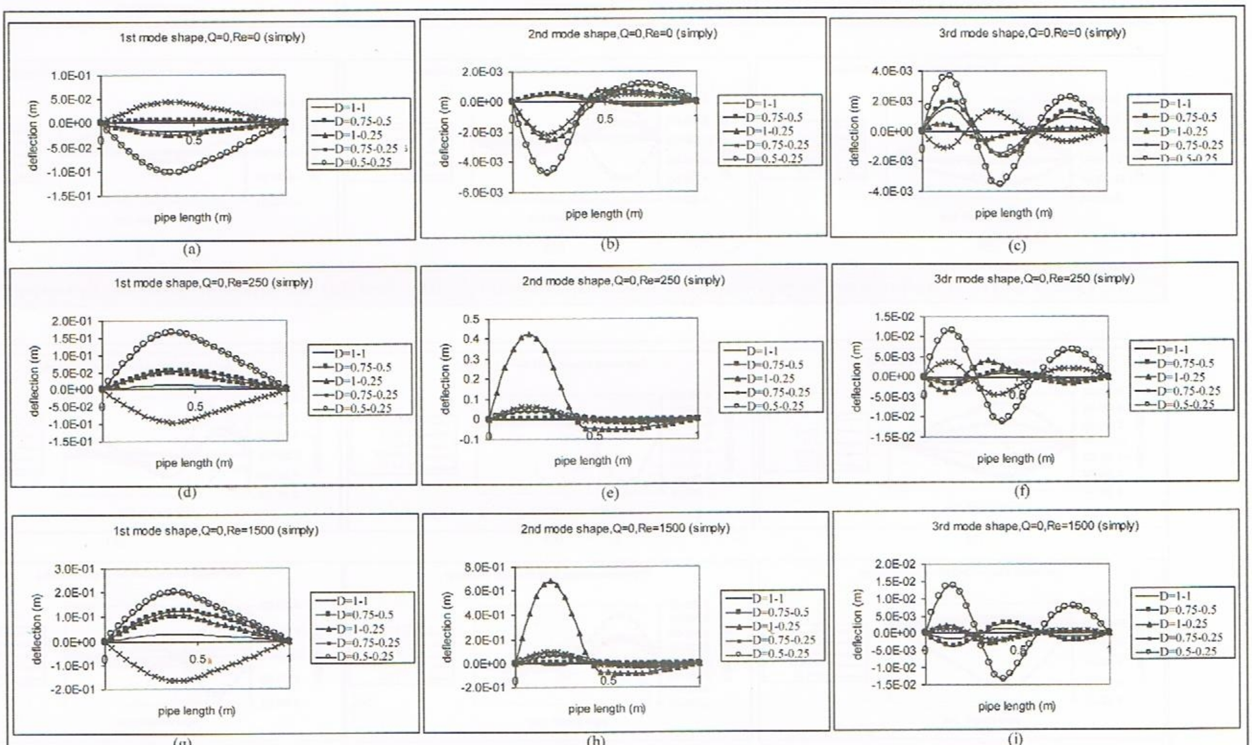


Fig.(3) The mode shapes for pipe conveying fluid with different velocities and pipe diameters and without heat flux (simply support)

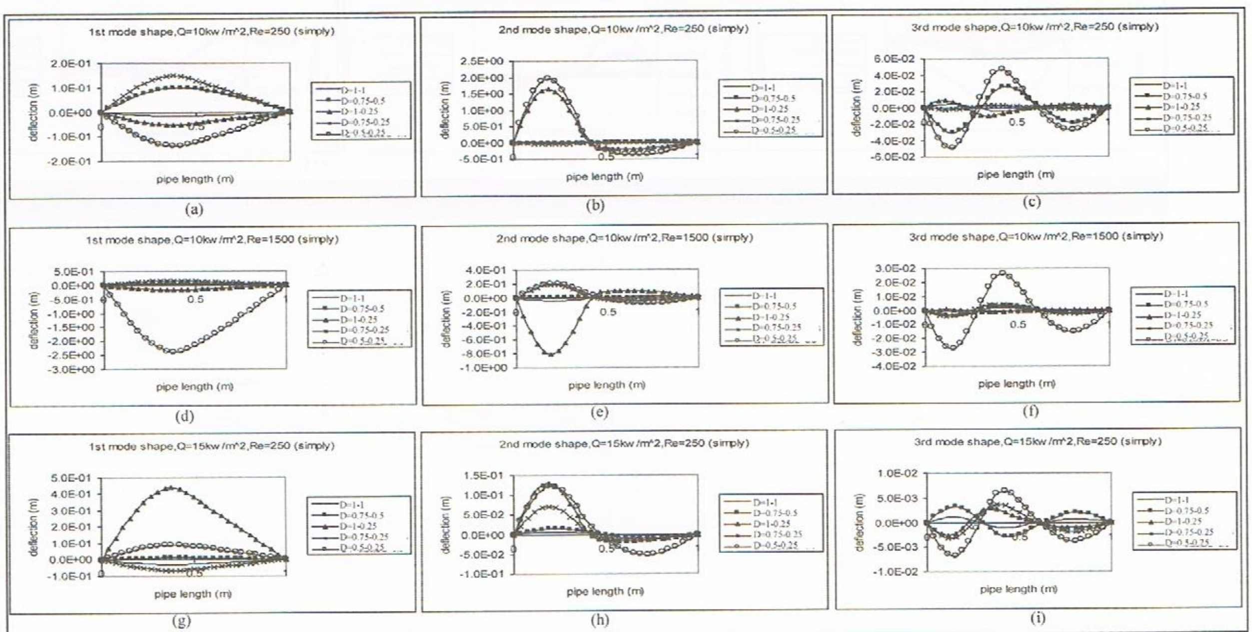
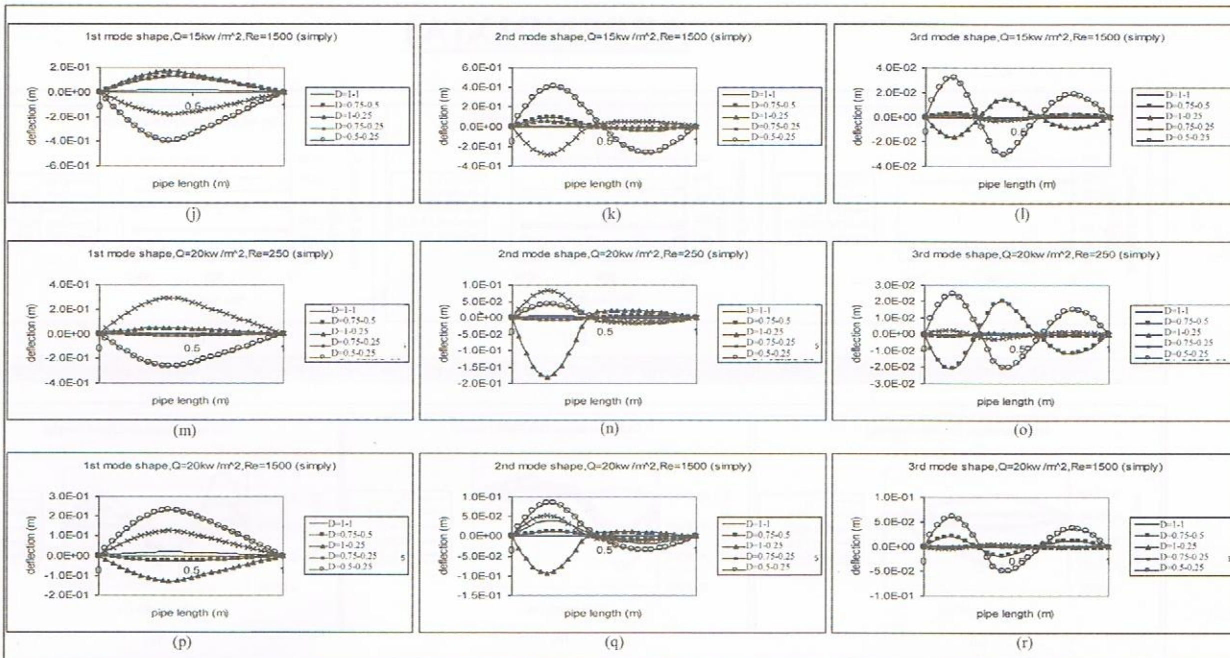


Fig.(4) The mode shapes for pipe conveying fluid with different velocities, heat flux and pipe diameters (simply support)



Cont. Fig.(4) The mode shapes for pipe conveying fluid with different velocities, heat flux and pipe diameters (simply support)

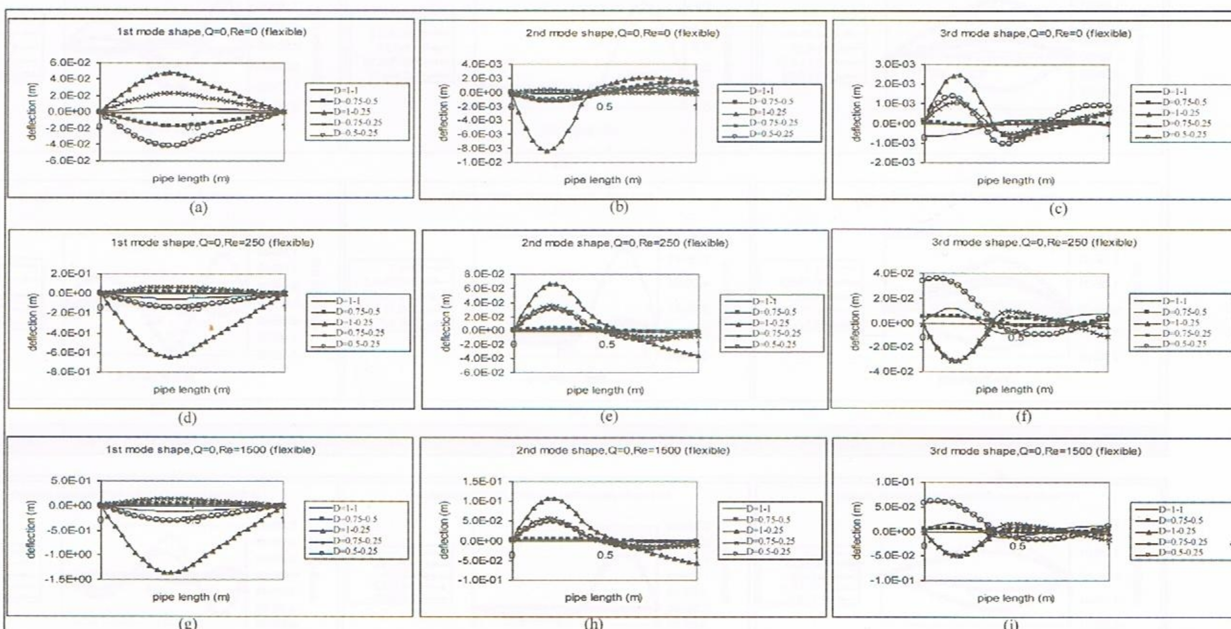


Fig.(5) Mode shapes for pipe conveying fluid with different velocities and pipe diameters and without heat flux (flexible support)

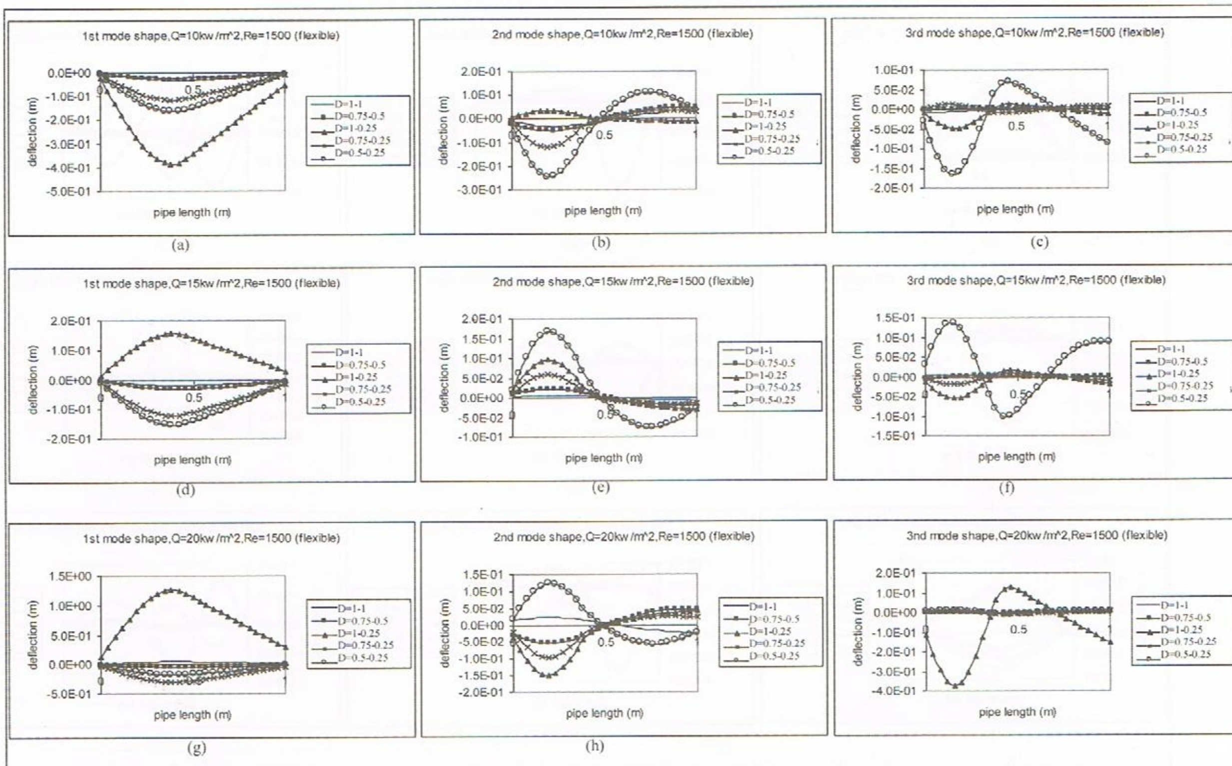


Fig.(6) The mode shapes for pipe conveying fluid with different velocities, heat flux and pipe diameters (flexible support)

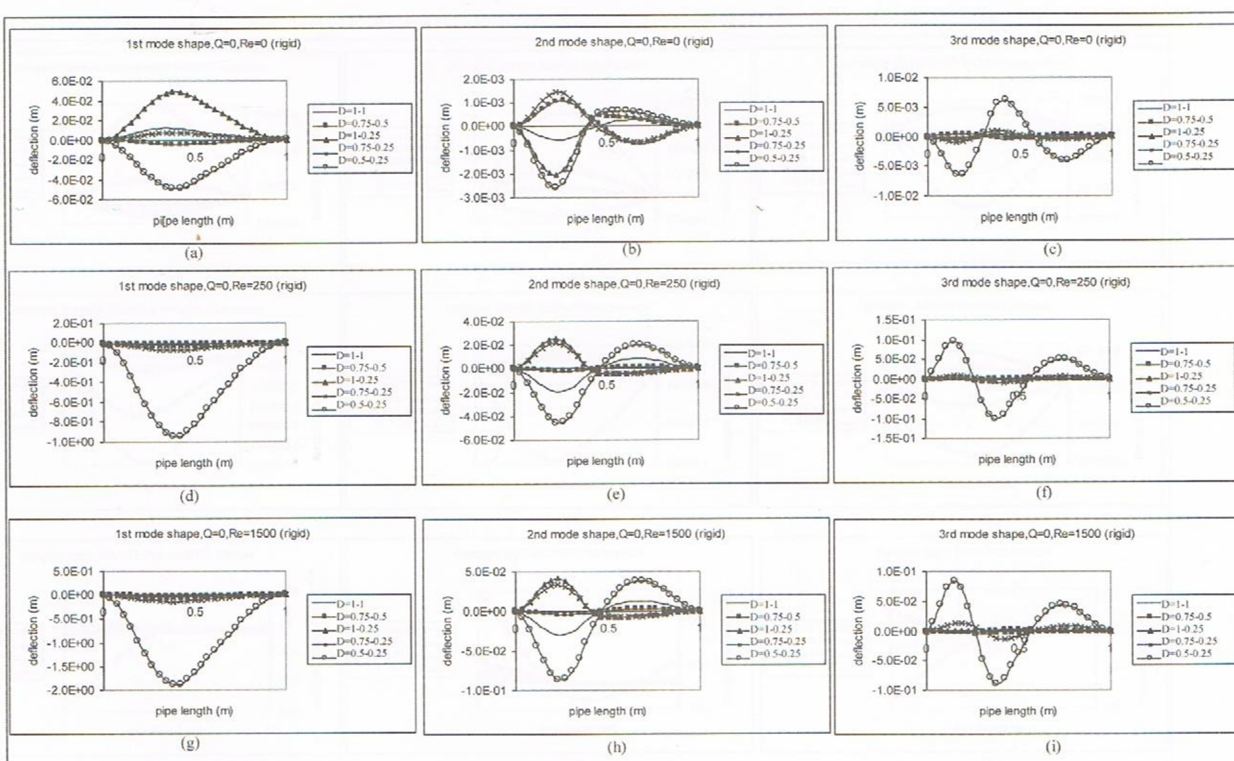


Fig.(7) The mode shapes for pipe conveying fluid with different velocities and pipe diameters and without heat flux (rigid support)

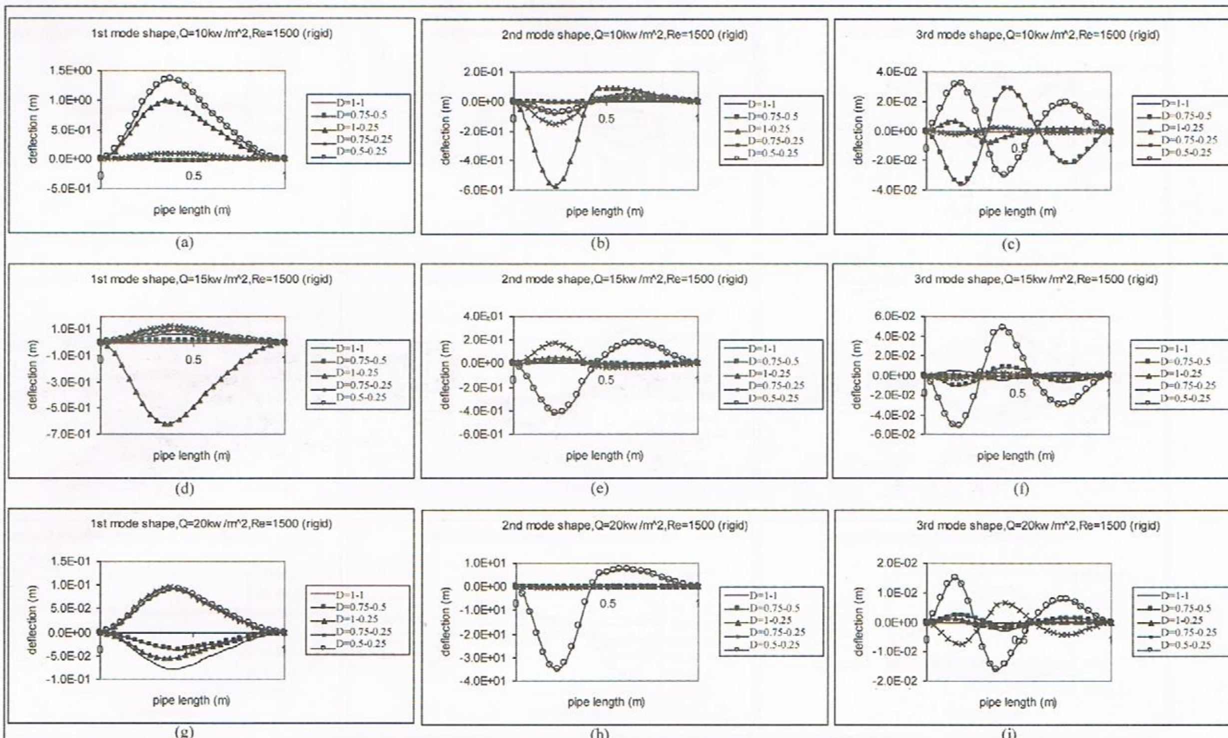


Fig.(8) The mode shapes for pipe conveying fluid with different velocities, heat flux and pipe diameters (rigid support)

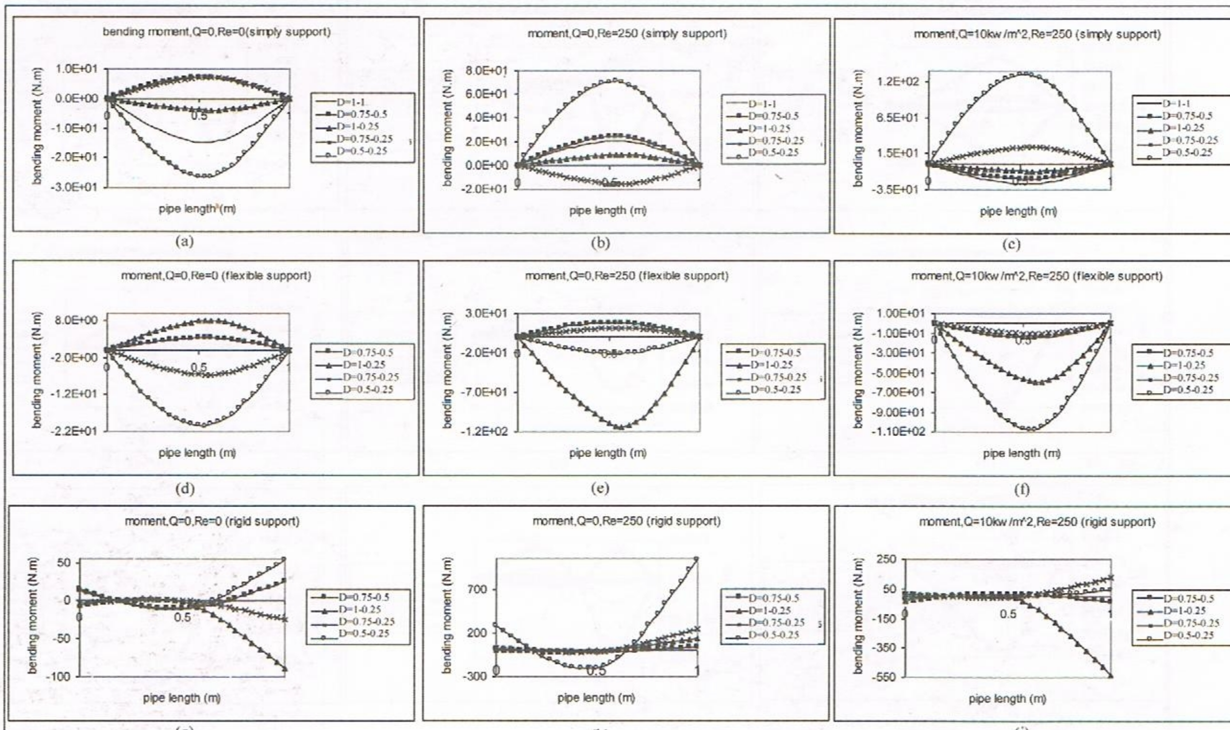


Fig.(9) Comparison the bending moment for different supports pipe conveying fluid at first natural frequencies and different velocities, heat flux and pipe diameters

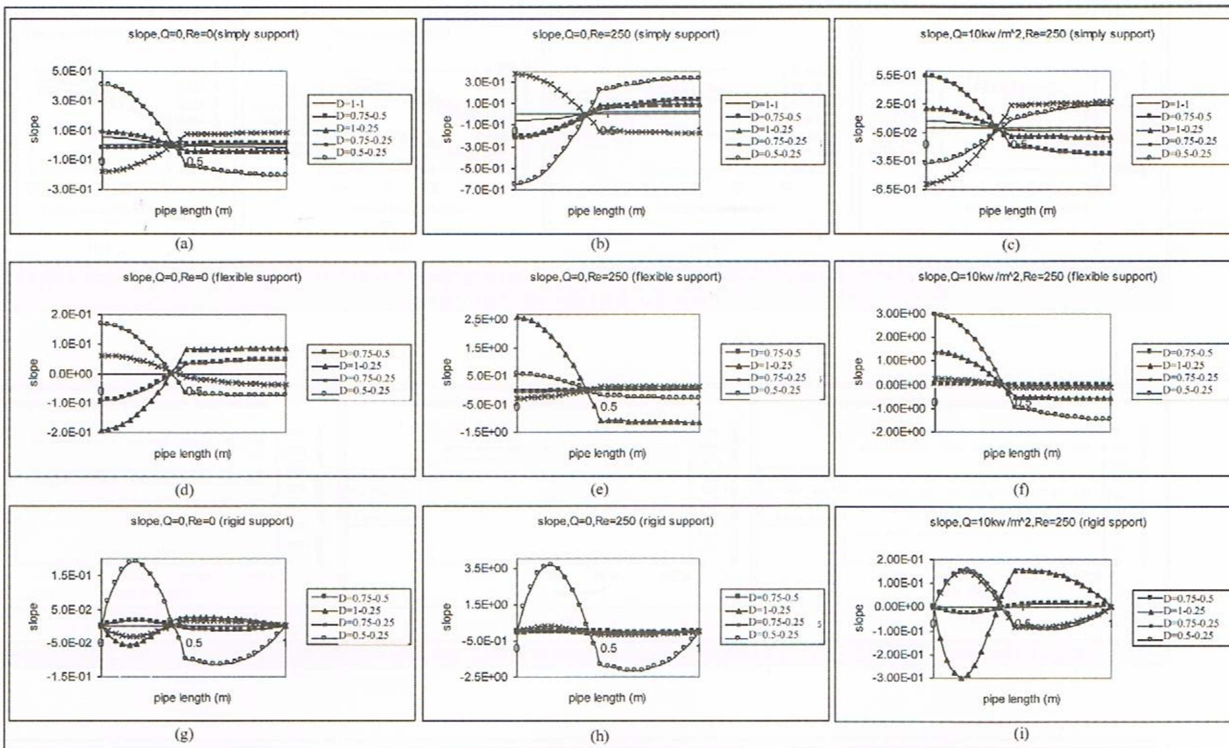


Fig.(10) Comparison slope for different supports pipe conveying fluid at first natural frequencies and different velocities, heat flux and pipe diameters

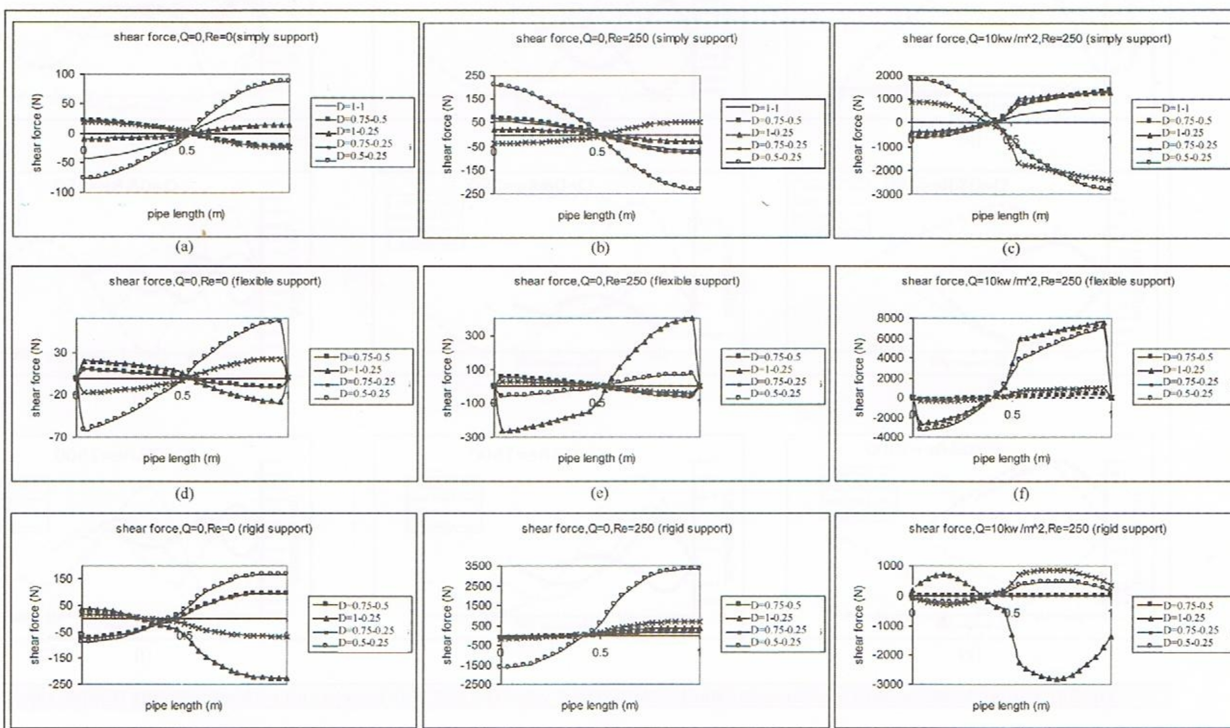


Fig.(11) Comparison shear force for different supports pipe conveying fluid at first natural frequencies and different velocities, heat flux and pipe diameters

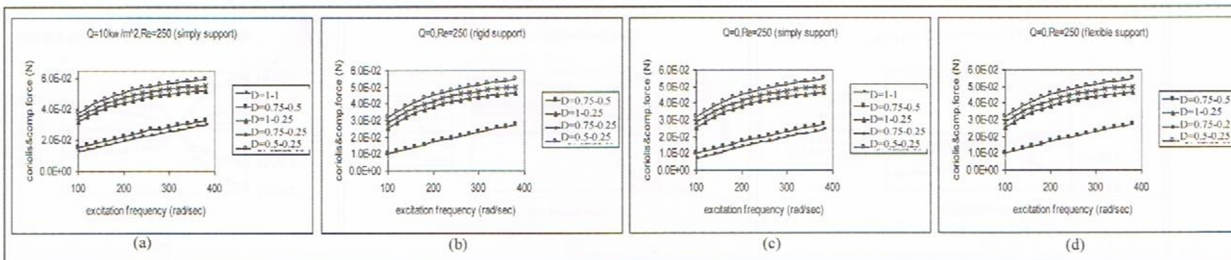


Fig.(12) Coriolis & compressive force for different supported pipe conveying fluid due to forced vibration at mid length with various velocities, heat flux and pipe diameters

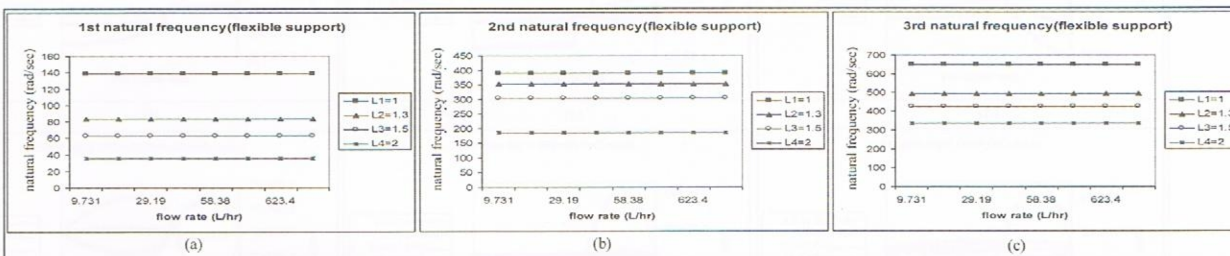


Fig.(13) Natural frequencies for pipe conveying fluid without heat flux and with various flow rates and various pipe lengths

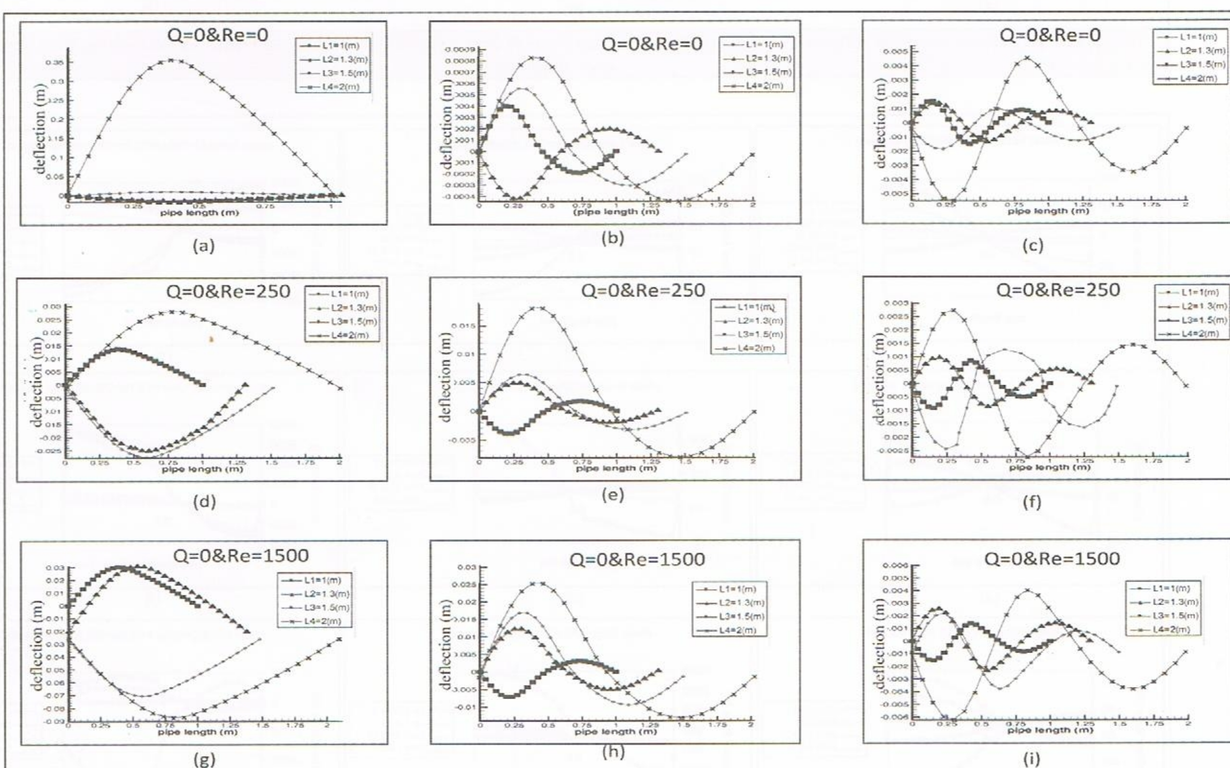


Fig.(14) The mode shapes for pipe conveying fluid with different velocities and pipe lengths and without heat flux (simply support)

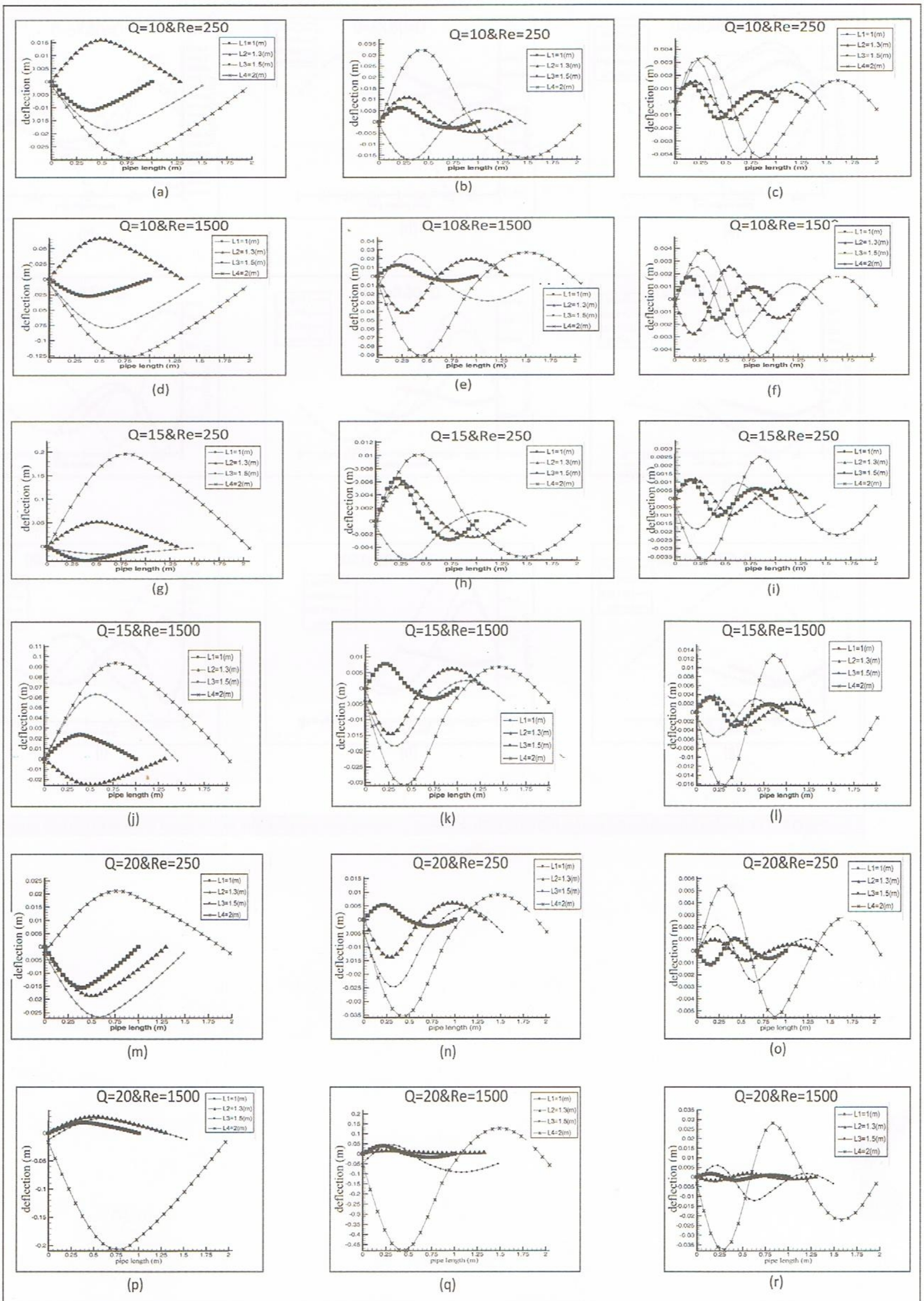


Fig.(15) The mode shapes for pipe conveying fluid with different velocities and pipe lengths and with different heat flux (simply support)

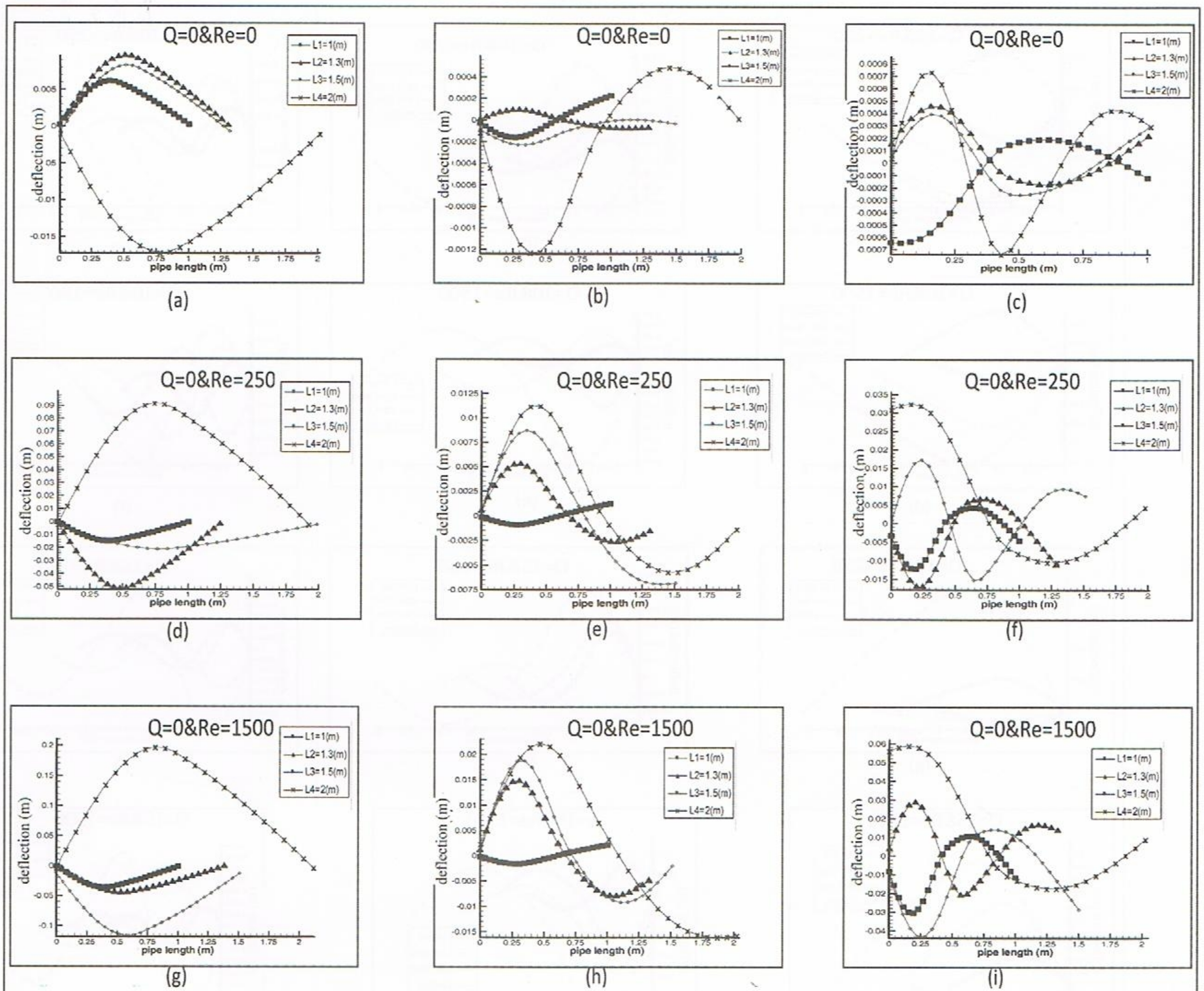


Fig.(16) The mode shapes for pipe conveying fluid with different velocities and pipe lengths and without heat flux (flexible support)

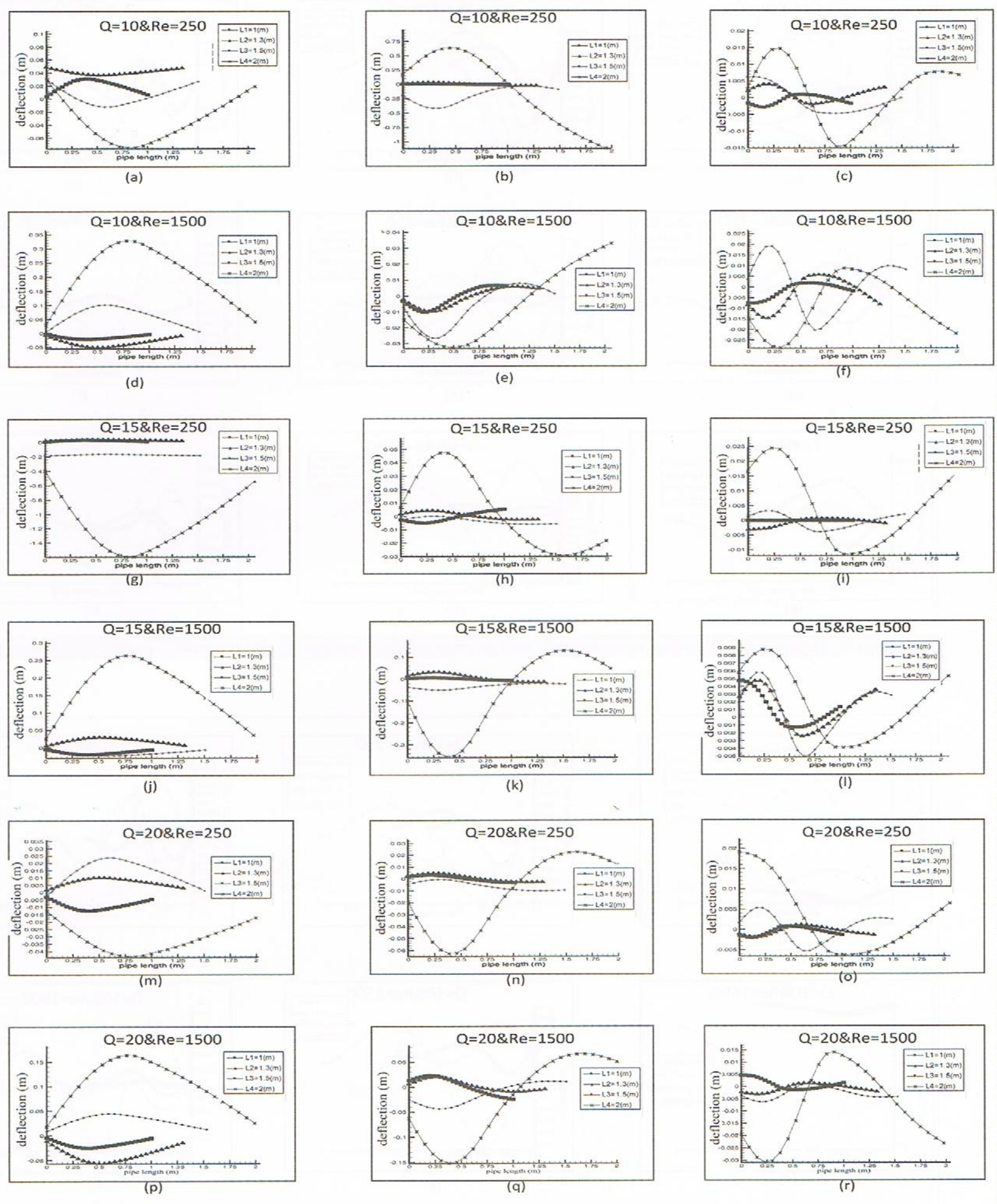


Fig.(17) The mode shapes for pipe conveying fluid with different velocities and pipe lengths and with different heat flux (flexible support)

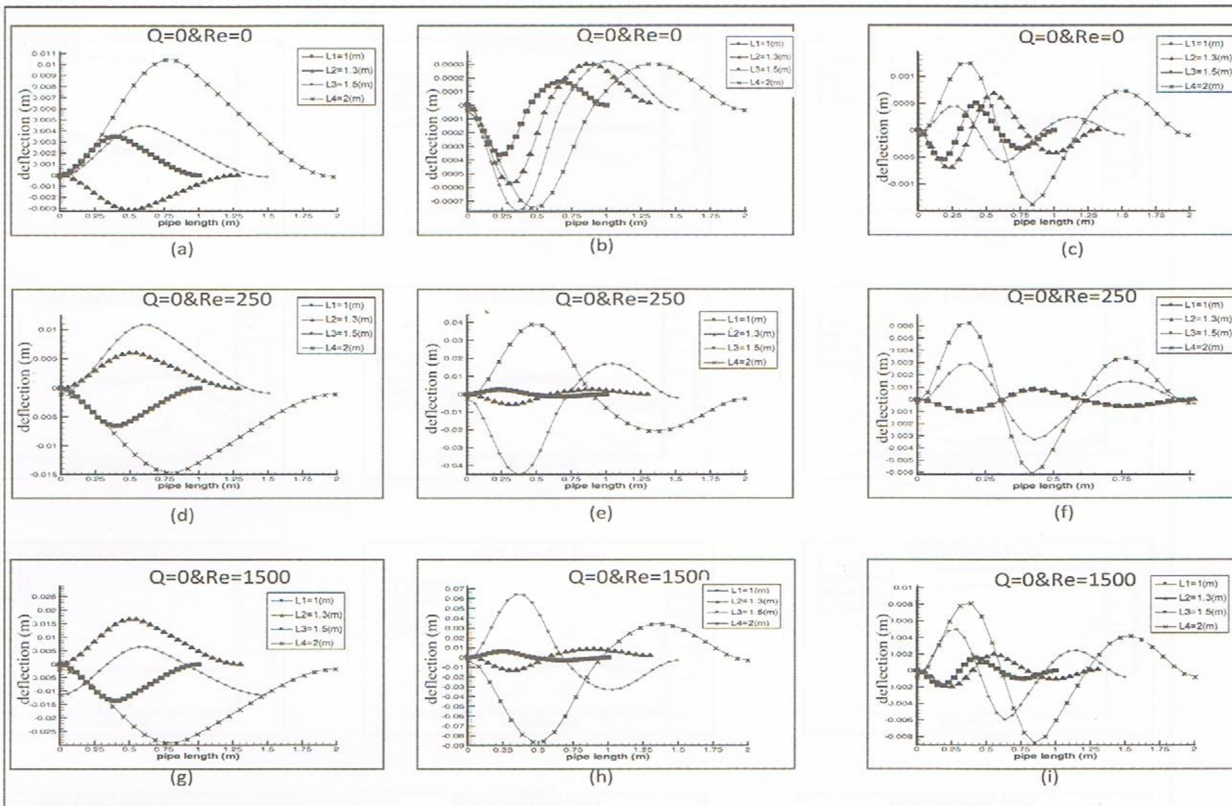


Fig.(18) The mode shapes for pipe conveying fluid with different velocities and pipe lengths and without heat flux (rigid support)

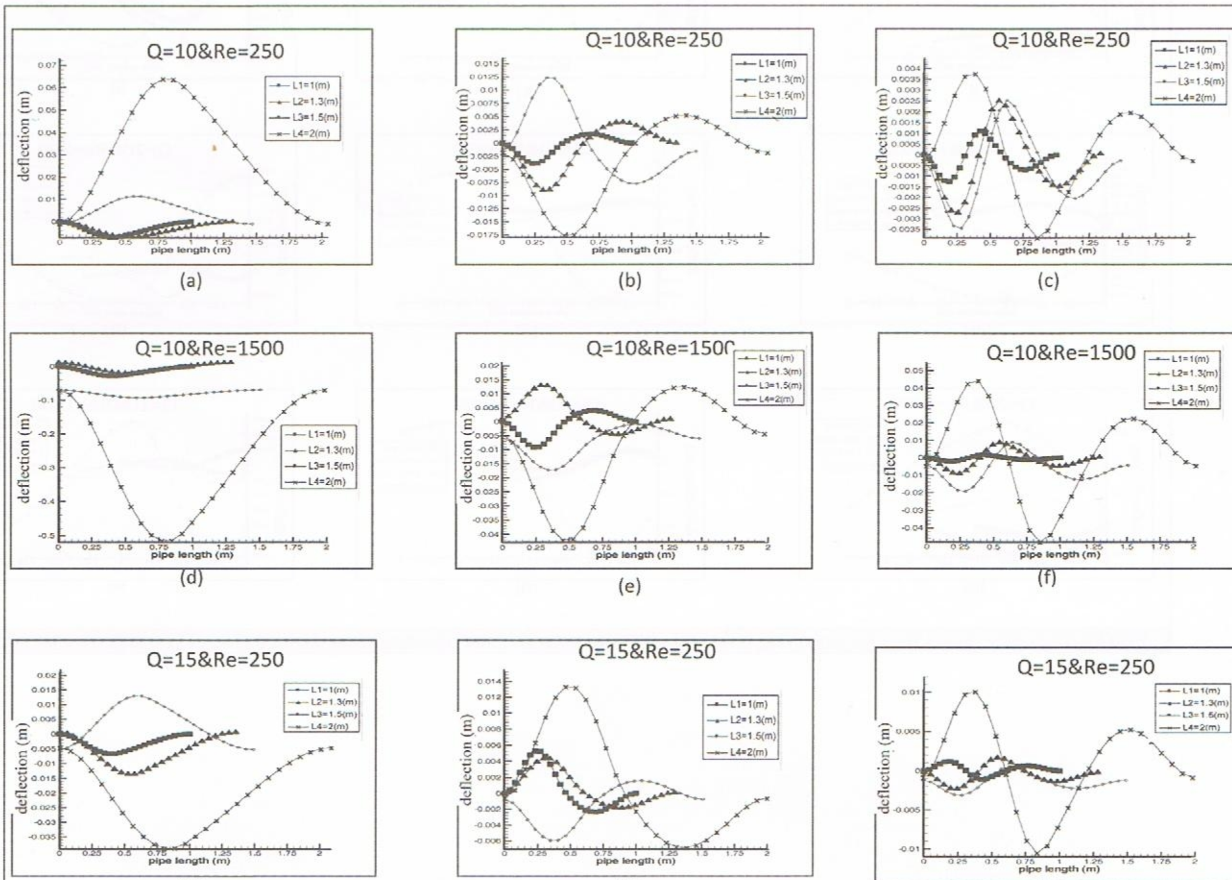
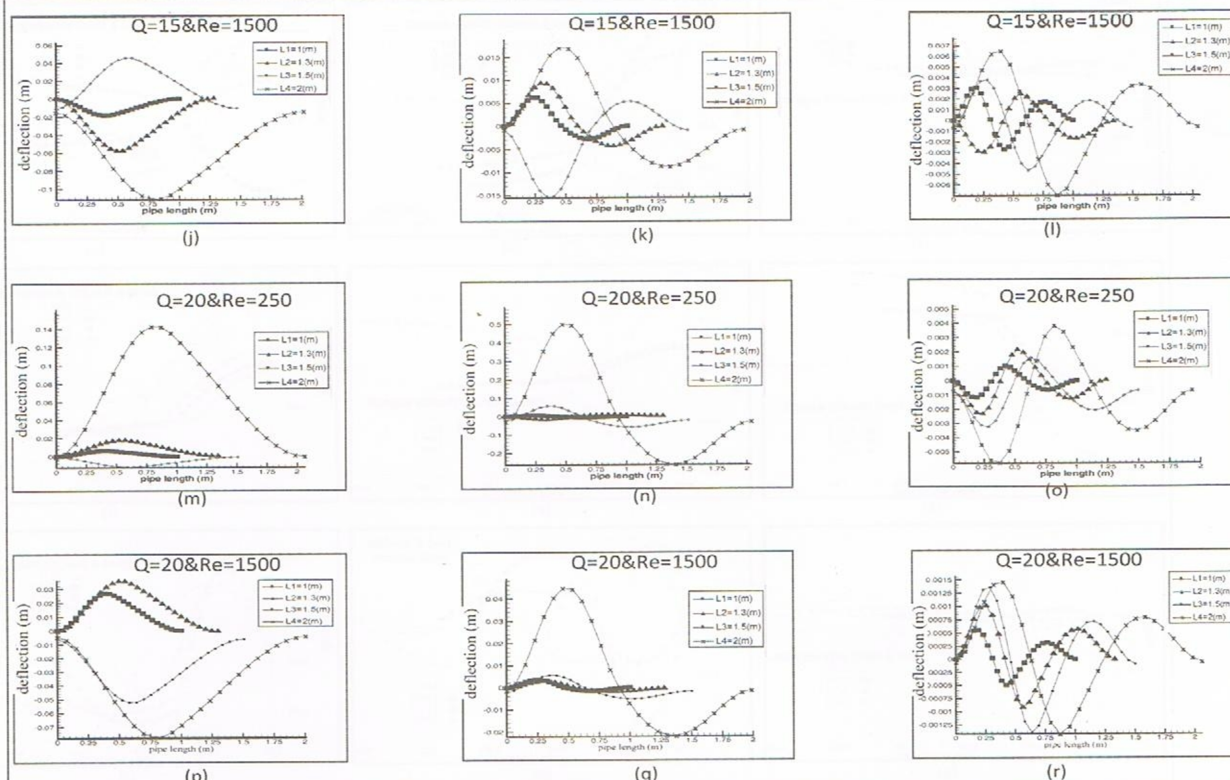


Fig (19) The mode shapes for pipe conveying fluid with different velocities and pipe lengths and with different heat flux (rigid support)



Cont. Fig (19) The mode shapes for pipe conveying fluid with different velocities and pipe lengths and with different heat flux (rigid support)

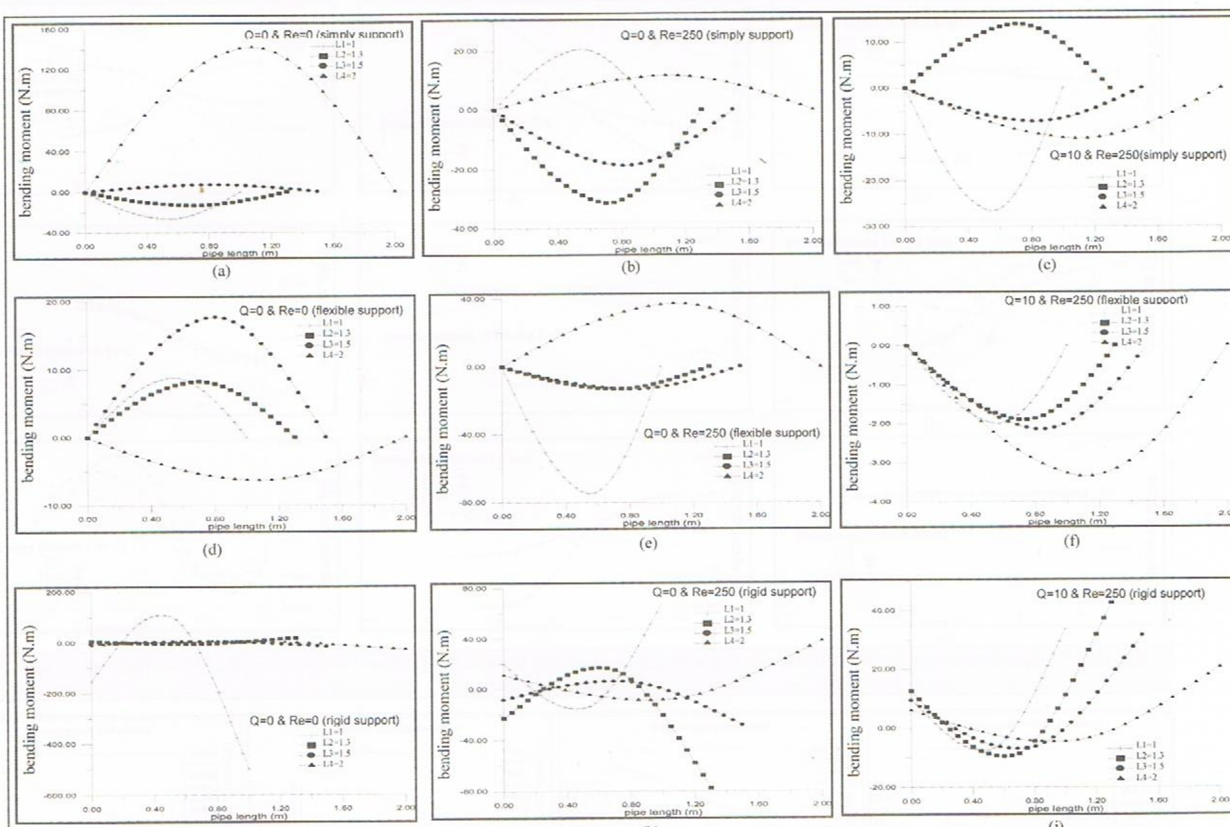


Fig.(20) Comparison bending moment for different supports pipe conveying fluid at first natural frequencies and different velocities, heat flux and pipe lengths

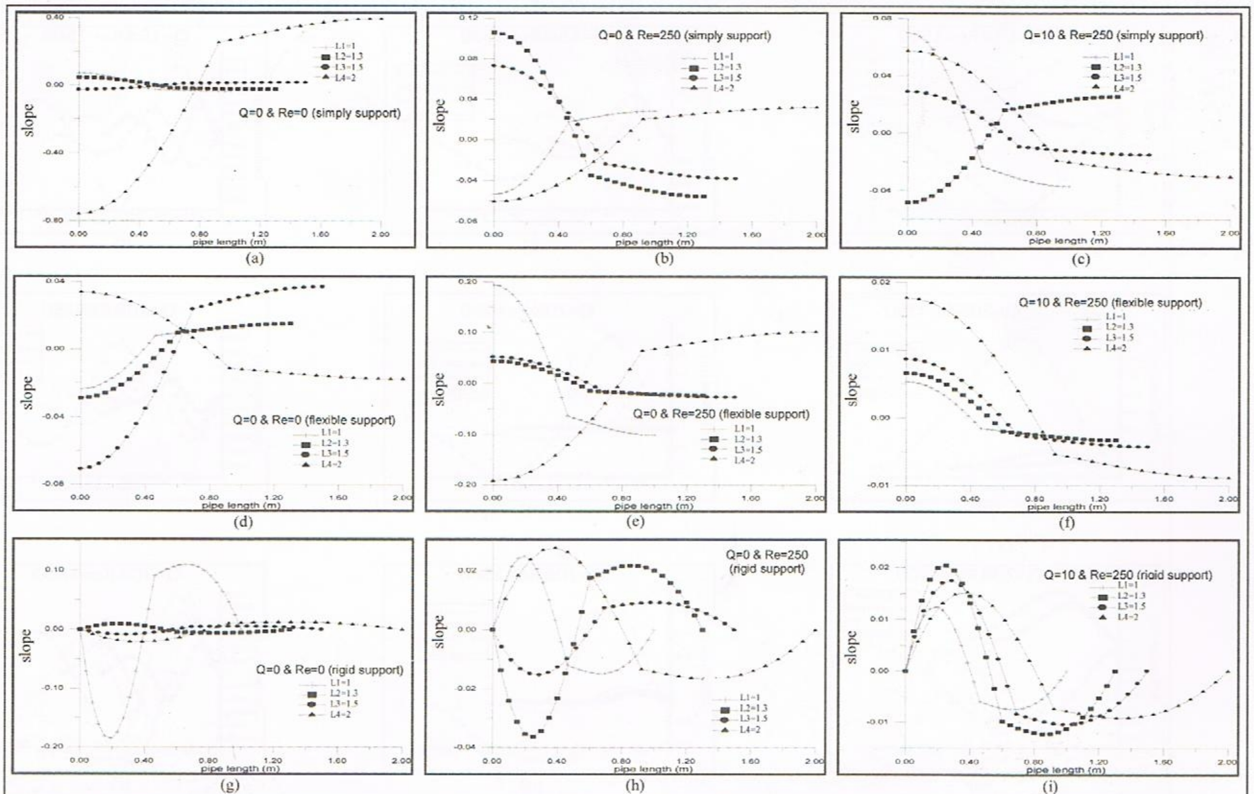


Fig.(21) Comparison slope for different supports pipe conveying fluid at first natural frequencies & different velocities, heat flux and pipe lengths

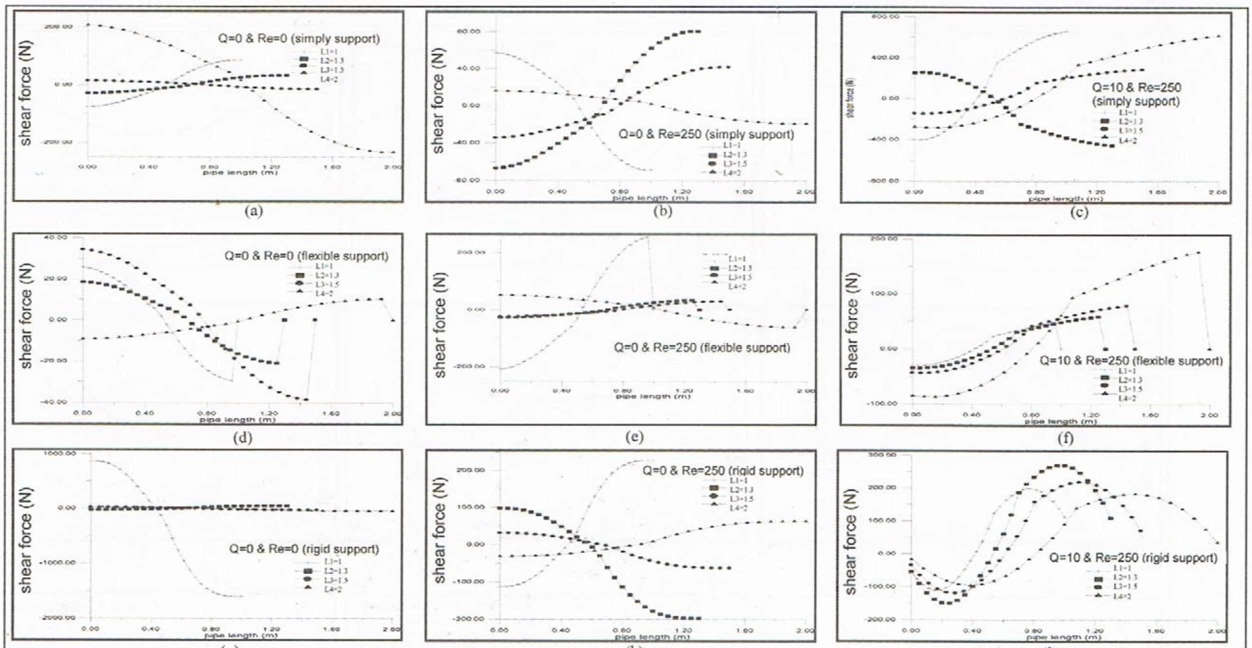


Fig. (22) Comparison shear force for different supports pipe conveying fluid at first natural frequencies &different velocities, heat flux & pipe lengths

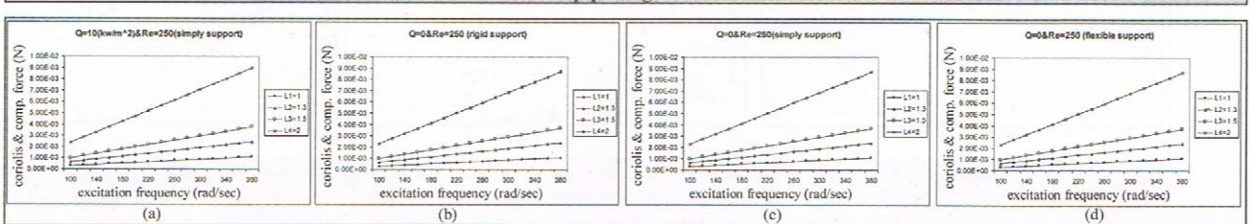


Fig.(23) Coriolis &compressive force for different supported pipe conveying fluid at mid length with various velocities, heat flux & pipe lengths

APPENDIX(B)
TABLES

I) FOR CHANGING DIAMETERS:

1) NO FLUID&NO HEAT

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	163	168	445	162.5	166	394.5	65	65.5	354	71	71	274.5	76.5	77	209
Ω_2	475.5	888.5	1352.5	530	736	1136	451	505	755	462.5	490	732.5	408	419	640.5
Ω_3	731.5	1768	2401	896	1571	2128.5	538.5	1352.5	1780	589	1102	1512.5	692	833	1141.5

2) FLUID&NO HEAT

Re=250

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	142	145.5	387	140	142	346	55.5	55.5	307	63	63.5	247	71.5	71.5	195
Ω_2	457.5	772	1179.5	392	646	997	382	473.5	701.5	422.5	459.5	685	383	394	602
Ω_3	652	1542	2071	629	1382.5	1873.5	487	1220	1636	520	998.5	1376	635.5	780	1069

Re=500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	142	145.5	387	140	142	346	55.5	55.5	307	63	63.5	247	71.5	71.5	195
Ω_2	457.5	772	1179.5	392	646	997	382	473.5	701.5	422.5	459.5	685	383	394	602
Ω_3	652	1542	2071	629	1382.5	1873.5	487	1220	1636	520	998.5	1376	635.5	780	1069

Re=750

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	142	145.5	387	140	142	346	55.5	55.5	307	63	63.5	247	71.5	71.5	195
Ω_2	457.5	772	1179.5	392	646	997	382	473.5	701.5	422.5	459.5	685	383	394	602
Ω_3	652	1542	2071	629	1382.5	1873.5	487	1220	1636	520	998.5	1376	635.5	780	1069

Re=1000

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	142	145.5	387	140	142	346	55.5	55.5	307	63	63.5	247	71.5	71.5	195
Ω_2	457.5	772	1179.5	392	646	997	382	473.5	701.5	422.5	459.5	685	383	394	602
Ω_3	652	1542	2071	629	1382.5	1873.5	487	1220	1636	520	998.5	1376	635.5	780	1069

Re=1500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	142	145.5	387	140	142	346	55.5	55.5	307	63	63.5	247	71.5	71.5	195
Ω_2	457.5	772	1179.5	392	646	997	382	473.5	701.5	422.5	459.5	685	383	394	602
Ω_3	652	1542	2071	629	1382.5	1873.5	487	1220	1636	520	998.5	1376	635.5	780	1069

3) FLUID&HEAT

a) Q=10 (kw/m^2)

Re=250

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	123.5	144.5	383	128	141	343	50	55.5	304	59	63	245.5	68.5	71.5	194.5
Ω_2	428	764.5	1169	450.5	641.5	989.5	398	470.5	697.5	407	457.5	682	366	393	600
Ω_3	660.5	1527.5	2052	864.5	1372.5	1860.5	502	1210.5	1624.5	550.5	993	1368.5	631.5	777.5	1065.5

Re=500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	124.5	144.5	383.5	129	141	343.5	50.5	55.5	304.5	59.5	63	245.5	69	71.5	194.5
Ω_2	424	765.5	1170	451	642	990.5	396	471	697.5	408.5	458	682.5	369	393.5	600.5
Ω_3	658.5	1529.5	2054.5	861.5	1374	1862	497	1212	1626.5	544.5	994	1370	632	778.5	1067

Re=750

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	125	144.5	383.5	129.5	141	344	50.5	55.5	305	59.5	63	246	69	71.5	194.5
Ω_2	423	765.5	1170.5	451	642.5	991	395.5	471	698	409	458	683	370	393.5	600.5
Ω_3	657.5	1530	2055	860.5	1374.5	1863	495	1212.5	1627	542.5	994.5	1370.5	632	778.5	1067

Re=1000

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	125	144.5	384	129.5	141	344	51	55.5	305	59.5	63	246	69	71.5	194.5
Ω_2	422	766	1170.5	451	642.5	991	395	471	698	409	458	683	370.5	393.5	601
Ω_3	657	1530	2055.5	860	1374.5	1863	494	1213	1627.5	541.5	995	1371	632	779	1067.5

Re=1500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	125.5	144.5	384	130	141	344	51	55.5	305	60	63	246	69.5	71.5	194.5
Ω_2	421.5	766	1171	451	642.5	991.5	394.5	471	698	409.5	458	683	371	393.5	601
Ω_3	657	1530.5	2056	859.5	1375	1863.5	493.5	1213.5	1627.5	540.5	995	1371.5	632	779	1067.5

b) Q=15 (kw/m^2)

Re=250

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	117	143.5	380.5	122.5	140	341	48	55	302	57	63	244	67	71	193.5
Ω_2	421.5	759	1160.5	445.5	637.5	983.5	398.5	468	693.5	401	455.5	679.5	359.5	391.5	598
Ω_3	657	1516.5	2037	891	1364.5	1849.5	512	1203.5	1615.5	560.5	988	1361.5	628.5	775	1062

Re=500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	118.5	143.5	381	124	140	342	48.5	55	303	58	63	244.5	68	71.5	194
Ω_2	428	760.5	1162.5	446	638.5	985.5	397	468.5	694.5	402.5	456	680	363	392	599
Ω_3	664	1519.5	2041	887.5	1367	1852.5	504.5	1206	1618	553.5	990	1364.5	629	776	1063.5

Re=750

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	119	143.5	381.5	124.5	140	342	49	55	303	58	63	245	68	71.5	194
Ω_2	427	761	1163	446.5	639	986	396.5	468.5	694.5	403	456.5	680.5	364.5	392.5	599
Ω_3	663	1520	2042	886	1367.5	1853.5	502	1206.5	1619	550.5	990.5	1365.5	629.5	776.5	1064.5

Re=1000

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	119	144	381.5	124.5	141	342	49	55	303.5	58	63	245	68	71.5	194
Ω_2	426.5	761	1163.5	446.5	639.5	986.5	396	468.5	694.5	403.5	456.5	680.5	365	392.5	599.5
Ω_3	662.5	1520.5	2042.5	885.5	1368	1854	500.5	1207	1619.5	549	991	1366	629.5	777	1064.5

Re=1500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	119.5	144	381.5	125	141	342	49	55	303.5	58.5	63	245	68.5	71.5	194
Ω_2	425.5	761	1163.5	446.5	639.5	986.5	395.5	468.5	694.5	403.5	456.5	680.5	365.5	392.5	599.5
Ω_3	661.5	1521	2043	885	1368	1854.5	499.5	1207.5	1619.5	548	991.5	1366.5	629.5	777	1065

c) Q=20 (kw/m^2)

Re=250

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	112	143	377	118	139.5	339	46.5	54.5	299.5	55.5	62.5	242.5	66	71	193
Ω_2	425.5	753	1151.5	440.5	633.5	977	397	465.5	690	395.5	453.5	676.5	353.5	390	596
Ω_3	661.5	1505	2021	914.5	1356	1838	522.5	1195.5	1605	568.5	982.5	1354.5	625	772	1058

Re=500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	113	143	378.5	119.5	139.5	340	47	55	301	56.5	62.5	243.5	67	71	193.5
Ω_2	433.5	755	1154.5	441.5	635	980	395.5	466	690.5	397	454.5	677.5	357.5	391	597
Ω_3	670	1508.5	2026	910	1359	1842	513	1199	1609	560.5	985.5	1358.5	626	774	1060.5

Re=750

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	113.5	143	378.5	120	139.5	340	47.5	55	301.5	56.5	62.5	244	67	71	193.5

Ω_2	432	755.5	1155	441.5	635.5	980.5	395	466	691	398	454.5	677.5	359	391.5	597.5
Ω_3	668.5	1509.5	2027.5	909	1360	1843.5	510	1199.5	1610	557.5	986.5	1360	626.5	774.5	1061.5

Re=1000

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	114	143	379	120	140	340.5	47.5	55	301.5	57	62.5	244	67	71	193.5
Ω_2	431.5	756	1155.5	442	636	981	394.5	466.5	691	398	454.5	678	360	391.5	193.5
Ω_3	668	1510	2028.5	908.5	1360.5	1844	508	1200.5	1610.5	555.5	987	1360.5	626.5	775	1062

Re=1500

Ω_n	D(1.0 -0.5)			D(0.75-0.5)			D(1.0-0.25)			D(0.75-0.25)			D(0.5-0.25)		
	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid	Flex.	Simp.	Rigid
Ω_1	114	143	379	120.5	140	340.5	47.5	55	302	57	62.5	244	67.5	71	193.5
Ω_2	430.5	756	1156	442	636	981.5	394.5	466.5	691	398.5	454.5	678	360.5	391.5	598
Ω_3	667	1510.5	2029	907.5	1361	1844.5	506.5	1201	1611	554	987.5	1361	626.5	775	1062.5

II) FOR CHANGING LENGTHS:

1) NO FLUID&NO HEAT

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω1	163	168	445	98	99.5	264	74	74.5	198	42	42	111.5
Ω2	475.5	888.5	1352.5	421.5	526.5	804	357.5	396	605	216.5	223	341
Ω3	731.5	1768	2401	580.5	1049.5	1431	510	789	1078	397.5	444.5	608.5

2)FLUID&NO HEAT

Re=250

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	142	145.5	387	83.5	84.5	225	63	63.5	169	35.5	35.5	95.5
Ω_2	457.5	772	1179.5	351.5	455	694	304	342	522.5	186.5	192.5	294.5
Ω_3	652	1542	2071	495.5	902	1230.5	426.5	678.5	926.5	336	382	523

Re=500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	142	145.5	387	83.5	84.5	225	63	63.5	169	35.5	35.5	95.5
Ω_2	457.5	772	1179.5	351.5	455	694	304	342	522.5	186.5	192.5	294.5
Ω_3	652	1542	2071	495.5	902	1230.5	426.5	678.5	926.5	336	382	523

Re=750

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	142	145.5	387	83.5	84.5	225	63	63.5	169	35.5	35.5	95.5
Ω_2	457.5	772	1179.5	351.5	455	694	304	342	522.5	186.5	192.5	294.5
Ω_3	652	1542	2071	495.5	902	1230.5	426.5	678.5	926.5	336	382	523

Re=1000

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	142	145.5	387	83.5	84.5	225	63	63.5	169	35.5	35.5	95.5
Ω_2	457.5	772	1179.5	351.5	455	694	304	342	522.5	186.5	192.5	294.5
Ω_3	652	1542	2071	495.5	902	1230.5	426.5	678.5	926.5	336	382	523

Re=1500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	142	145.5	387	83.5	84.5	225	63	63.5	169	35.5	35.5	95.5
Ω_2	457.5	772	1179.5	351.5	455	694	304	342	522.5	186.5	192.5	294.5
Ω_3	652	1542	2071	495.5	902	1230.5	426.5	678.5	926.5	336	382	523

3) FLUID&HEAT

a)Q=10 (kw/m^2)

Re=250

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	123.5	144.5	383	74	83.5	223	56.5	63	167.5	32.5	35.5	94
Ω_2	428	764.5	1169	340	450.5	687.5	280	338.5	517	171	190.5	291
Ω_3	660.5	1527.5	2052	519.5	893	1218	453	671	917	318	377.5	517

Re=500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	124.5	144.5	383.5	75	83.5	223	57	63	167.5	33	35.5	94.5
Ω_2	424	765.5	1170	341	451.5	688.5	282	339	518	173	191	292
Ω_3	658.5	1529.5	2054.5	513.5	894.5	1220	448.5	672.5	918.5	320	378.5	518

Re=750

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	125	144.5	383.5	75	84	223.5	57.5	63	168	33	35.5	94.5
Ω_2	423	765.5	1170.5	341	451.5	688.5	283	339.5	518	173.5	191	292
Ω_3	657.5	1530	2055	511.5	895	1220.5	446.5	673	919	320.5	379	518.5

Re=1000

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	125	144.5	384	75	84	223.5	57.5	63	168	33	35.5	94.5
Ω_2	422	766	1170.5	341	451.5	689	283.5	339.5	518	174	191	292
Ω_3	657	1530	2055.5	510.5	895	1220.5	446	673	919	320.5	379	518.5

Re=1500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	125.5	144.5	384	75.5	84	223.5	57.5	63	168	33	35.5	94.5
Ω_2	421.5	766	1171	341.5	451.5	689	284	339.5	518.5	174.5	191	292

Ω_3	657	1530.5	2056	509.5	895	1221	445	673	919.5	321	379	519

b)Q=15 (kw/m^2)

Re=250

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	117	143.5	380.5	75.5	84	223.5	57.5	63	168	31	35	93
Ω_2	421.5	759	1160.5	341.5	451.5	689	284	339.5	518.5	165.5	188.5	288.5
Ω_3	657	1516.5	2037	509.5	895	1221	445	673	919.5	311.5	374.5	512.5

Re=500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	118.5	143.5	381	71.5	83	221.5	55	62.5	166.5	31.5	35	93.5
Ω_2	428	760.5	1162.5	335.5	448	683.5	274.5	337	514.5	168	189.5	289.5
Ω_3	664	1519.5	2041	520	888.5	1211.5	452.5	668	912	313.5	376	514.5

Re=750

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	119	143.5	381.5	72	83	222	55	62.5	166.5	32	35	94
Ω_2	427	761	1163	335.5	448.5	684	275.5	337	514.5	168.5	189.5	290
Ω_3	663	1520	2042	517.5	889	1212.5	450.5	668.5	913	314.5	376.5	515

Re=1000

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	119	144	381.5	72	83	222	55	62.5	166.5	32	35	94
Ω_2	426.5	761	1163.5	336	448.5	684.5	276	337	515	169	190	290
Ω_3	662.5	1520.5	2042.5	516.5	889.5	1213	449.5	668.5	913.5	315	376.5	515.5

Re=1500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	119.5	144	381.5	72	83.5	222	55.5	62.5	167	32	35	94
Ω_2	425.5	761	1163.5	336	449	684.5	276.5	337.5	515	169.5	190	290.5
Ω_3	661.5	1521	2043	515	889.5	1213.5	448.5	669	913.5	315	376.5	515.5

c)Q=20 (kw/m^2)

Re=250

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	112	143	377	72	83.5	222	55.5	62.5	167	30	34.5	92
Ω_2	425.5	753	1151.5	336	449	684.5	276.5	337.5	515	161	187	286
Ω_3	661.5	1505	2021	515	889.5	1213.5	448.5	669	913.5	306	370.5	507.5

Re=500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	113	143	378.5	69	82.5	220	53	62	165	30.5	35	93
Ω_2	433.5	755	1154.5	330.5	445	679	268.5	334.5	510.5	163.5	188	287.5
Ω_3	670	1508.5	2026	525	882	1203	454	663	905	308.5	373	510.5

Re=750

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	113.5	143	378.5	69	82.5	220	53	62	165.5	31	35	93
Ω_2	432	755.5	1155	330.5	445.5	679.5	269	334.5	511	164.5	188.5	288
Ω_3	668.5	1509.5	2027.5	522.5	882.5	1204	452.5	663.5	906.5	309	373.5	511

Re=1000

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	114	143	379	69	82.5	220.5	53	62	165.5	31	35	93
Ω_2	431.5	756	1155.5	330.5	445.5	679.5	269.5	335	511.5	164.5	188.5	288
Ω_3	668	1510	2028.5	521	883	1204.5	451.5	664	907	309.5	374	511.5

Re=1500

natural frequency	L(0.5-0.5)			L(0.65-0.65)			L(0.75-0.75)			L(1.0-1.0)		
	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid	flexible	simply	rigid
Ω_1	114	143	379	69.5	82.5	220.5	53.5	62	165.5	31	35	93
Ω_2	430.5	756	1156	331	445.5	680	270	335	511.5	165	188.5	288.5
Ω_3	667	1510.5	2029	520	883.5	1205	450.5	664.5	907.5	310	374	512