Performance Comparison of Two Estimators for Two-Phase Permanent Magnet Synchronous Motor

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Abstract

This paper presents and compares the performance of two Kalman filter schemes, the discrete extended Kalman filter (EKF) and unscented Kalman filter (UKF) for estimating the states (winding currents, rotor speed and rotor angular position) of two-phase Permanent Magnet Synchronous Motor (PMSM). Estimating the states of the system is performed by propagating the mean and covariance of the state distribution. For linear systems, the general recursive Kalman filter algorithm based on MMSE (minimum mean squared error) is the straightforward estimation technique to be implemented. For nonlinear systems, extended Kalman filter (EKF) is considered to be the best nonlinear estimator. The EKF is based on linearizing the state and output equations at every sampling instant. Therefore, this estimator requires continuously computation of the Jacobian matrix. The unscented Kalman filter (UKF) is based on implementation of the unscented transformation (UT) to the nonlinear state distribution (motor model). The UT uses the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation. Apply this intuition to motor model, a set or cloud of points are generated around each state of motor model with specified sample mean and sample covariance. The nonlinear function (PMSM model) is applied to each of these points in turn to yield a transformed sample, and the predicted mean and covariance are calculated from the transformed sample. Based on predicted mean and covariance the UKF recursive algorithm can be developed. The performance comparisons are based on standard deviation estimation errors of both

estimators and the time computation effort required execute the algorithms of both filters. The simulated results show that the UKF gives best estimates at motor low speed, while its estimation performance degrade at high motor speed. On the other hand, the EKF shows bad estimation characteristics at low frequency and it yields good estimates at high source frequency. However,, the EKF algorithm keeps lower time computation effort over wide range of rotor speed than that required to execute the UKF software for the same range of source frequency. The PMSM motor model and the algorithms of both filters are built in Matlab package using S-function capability and scalar control strategy are used to account for constant stator magnetizing flux.

Keywords: Two-phase Permanent Magnet Synchronous Motor, Scalar control, EKF, UKF.

Introduction:

In order to permit tractable algorithms for tracking and control applications, an approximate state estimate must be generated. The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown [1-3].

However, the standard Kalman filter addresses the problem of estimating the state of a controlled process that is governed by a linear stochastic difference equation. But many of the processes to be estimated and (or) the measurement relationship to the process is non-linear. Therefore, for the standard Kalman filter to be applied to such nonlinear systems the nonlinear system is linearized first and then the recursive equations of the standard Kalman filter are applied for time update. The Kalman filter which tackles the estimation problem of linearized nonlinear process and linearizes about the current mean and covariance is referred linearized Kalman filter or alternatively extended Kalman filter (EKF) [3].

Thus, the EKF is an estimation algorithm wherein linearizing the nonlinear system involves the calculation of Jacobians and substituting them for the linear transformation in the KF equations. However, the following major shortcomings have been reported for the EKF [4]

(1) The EKF is based on Jacobian calculation. This calculation is sometimes of extremely difficult and error-prone process.

(3) Linearization is based on the Taylor series approximation, considering only the first two terms of the series and ignoring the other higher order terms.
(2) Linearization can produce highly unstable filter performa- nce if the time step intervals are not sufficiently small.
(4) Sufficiently small time step intervals usually imply high computational overhead as the number of calculations demanded for the generation of the Jacobian and the predictions of state estimate and covariance are large

As the EKF totally depends on the linearization to propagate the mean and covariance, it is obvious that EKF would give bad estimates if any one of the above limitations is encountered. To overcome these limitations, the unscented transformation was derived which utilizes a more direct and simple approach to propagate the mean and covariance.

The main advantage of implementing unscented transformation on a nonlinear estimation problem is that it approximates the mean to third order, which is better than linearization, and it approximates the covariance to third order, which is the same as linearization. Therefore, loss of the higher-order terms can be avoided in the propagation of the state of the system by using the full nonlinear equations [3,4].

Figure (1) illustrates the mean and covariance propagation in all three transformations. The mean in the EKF and UT are similar to that of the true nonlinear transformation. But, when covariance propagation is considered, the UT outperforms the EKF. And moreover there is no need of calculating any Jacobians in the unscented transformation algorithm. The order of computational effort is almost the same in both algorithms. When the computational effort is same in both cases, the unscented transformation is preferred over the EKF for the better accuracy [4].Thus, the unscented transformation is proved to be more accurate in propagating the mean and covariance when compared with the linearization method used in EKF [3,4].

The objective of the work is to use the EKF and the UKF for estimating the states of a two-phase permanent magnet synchronous motor and then comparisons have been made between their estimation performances.

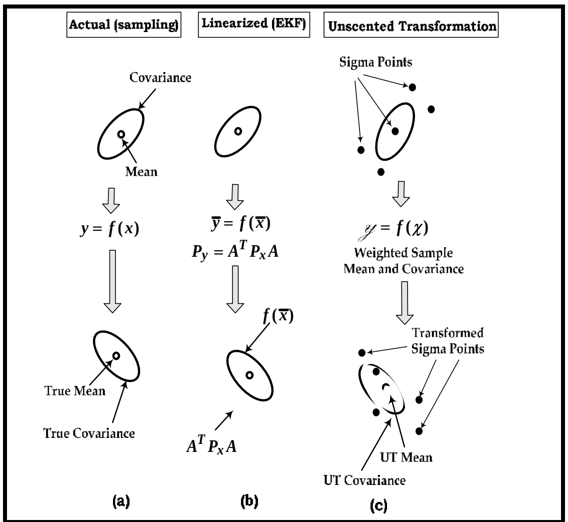

Figure (1): Mean and Covariance propagation in (a) Actual nonlinear transformation, (b) Extended Kalman filter (first order linearization), (c) Unscented Transformation

Development of Two-Phase PMSM State Space for Kalman Filter Estimators:

Figure (2) shows the schematic cross section of two phase synchronous machine shows having a permanent magnet rotor and two identical stator windings a and b whose axes are in quadrature. The salient-pole rotor is revolving at synchronous speed ω_{rm} , whose angular position is given by $\theta_{rm} = \omega_{rm}\, t$. The stator windings are connected to a balanced two-phase voltage source of frequency ω_s .

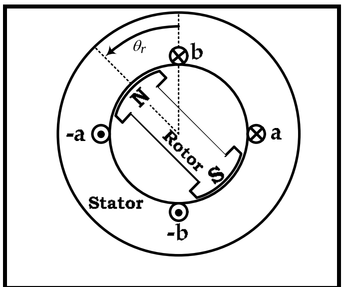

Figure (2) Schematic of two-phase salient-pole synchronous machine

Using Kirchhoff's voltage law, on can obtain [5,6]

$$u_{as} = r_s\, i_{as} + d\psi_{as} / dt \qquad (1)$$

$$u_{bs} = r_s\, i_{bs} + d\psi_{bs} / dt \qquad (2)$$

Here, u_{as} and u_{as} are the phase voltages in the stator windings *as* and *bs*; i_{as} and i_{as} are the phase currents in the stator windings; r_s is the resistance of the stator windings, ψ_{as} and ψ_{as} are the stator flux linkages, which can be expressed as

$$\psi_{as} = L_{asas}\, i_{as} + L_{asbs}\, i_{bs} + \psi_{asm} \qquad (3)$$

$$\psi_{bs} = L_{bsas}\, i_{as} + L_{bsbs}\, i_{bs} + \psi_{bsm} \qquad (4)$$

where L_{asas} and L_{bsbs} are the self-inductances of the stator winding which are given in terms of leakage inductance L_{ls} and magnetizing inductance L_m as

$$L_{ss} = L_{asas} = L_{bsbs} = L_{ls} + L_m \qquad (5)$$

Since the stator windings are displaced by 90 electrical degrees, hence, the mutual inductances between the stator windings are $L_{asbs} = L_{bsas} = 0$.

The flux linkages are periodic functions of the angular displacement (rotor position), and hence

$$\psi_{asm} = \psi_m \sin(\theta_{rm}) \qquad (6)$$

$$\psi_{bsm} = -\psi_m \cos(\theta_{rm}) \qquad (7)$$

Then, from Eq.(3) and (4), one can have

$$\psi_{as} = L_{ss}\, i_{as} + \psi_m \sin(\theta_{rm}) \qquad (8)$$

$$\psi_{bs} = L_{ss}\, i_{bs} - \psi_m \cos(\theta_{rm}) \qquad (9)$$

Therefore, one finds

$$u_{as} = r_s\, i_{as} + L_{ss}\; di_a / dt + \psi_m\; \omega_{rm} \cos\theta_{rm} \qquad (10)$$

$$u_{bs} = r_s\, i_{bs} + L_{ss}\; d\, i_{bs} / dt - \psi_m\; \omega_{rm} \sin\theta_{rm} \qquad (11)$$

Using Newton's second law

$$T_e - B_m\, \omega_{rm} - T_L = J \frac{d^2\theta_{rm}}{dt^2}$$

we have

$$\frac{d\omega_{rm}}{dt} = \frac{1}{J}\left(T_e - B_m\, \omega_{rm} - T_L\right) \qquad (12)$$

$$\frac{d\,\theta_{rm}}{dt} = \omega_{rm} \qquad (13)$$

where B_m is the of viscous friction coefficient that acts on the motor shaft and its load, J is the rotor moment of inertia and T_L is the external load. The expression for the electromagnetic torque developed by permanent-magnet motors can be obtained by using the coenergy

Then, one has

$$Wc = \frac{1}{2}(L_{ss} i_{as}^2 + L_{ss} i_{bs}^2) + \psi_m i_{as} \sin(\theta_{rm}) - \psi_m i_{bs} \cos(\theta_{rm}) \qquad (14)$$

$$T_e = \frac{\partial W_c}{\partial \theta_{rm}} = \frac{P\,\psi_m}{2}(i_{as} \cos\theta_{rm} + i_{bs} \sin\theta_{rm}) \qquad (15)$$

Augmenting the circuitry transients with the torsional-mechanical dynamics and accounting for uncertainties in load and noises that may corrupt input signal, the mathematical model of two-phase permanent-magnet motors can be written in the following form [6]:

$$\begin{aligned} \frac{di_{as}}{dt} &= -\frac{r_s}{L_{ss}} i_{as} - \frac{\psi_m}{L_{ss}} \omega_{rm} \cos\theta_{rm} + \frac{1}{L_{ss}}(u_{as} + \Delta u_{as}) \\ \frac{di_{bs}}{dt} &= -\frac{r_s}{L_{ss}} i_{bs} + \frac{\psi_m}{L_{ss}} \omega_{rm} \sin\theta_{rm} + \frac{1}{L_{ss}}(u_{bs} + \Delta u_{bs}) \end{aligned} \qquad (16)$$

$$\frac{d\omega_{rm}}{dt} = \frac{P\,\psi_m}{2\,J}(i_{as} \cos\theta_{rm} + i_{bs} \sin\theta_{rm}) - \frac{B_m}{J}\omega_{rm} - \frac{1}{J}(T_L + \Delta T_L)$$

$$\frac{d\theta_{rm}}{dt} = \omega_{rm}$$

where Δu_a and Δu_b are noise terms due to errors in u_a and u_b. $\Delta\alpha$ is a noise term due to uncertainty in the load torque.

It is assumed that the measurements of the two winding currents may be performed by sense resistors. The measurements are distorted by measurement noises Δi_{as} and Δi_{bs}, which are due to things like sense resistance uncertainty, electrical noise or quantization errors. Then, the noise corrupted measurements can be given by

$$y_1 = i_{as} + \Delta i_{as}\,; \qquad y_2 = i_{bs} + \Delta i_{bs}$$

Letting, $x_1 = i_{as}$, $x_2 = i_{bs}$, $x_3 = \omega_{rm}$, and $x_4 = \theta_{rm}$, the aforementioned dynamic equations can be simplified as

$$\begin{aligned} \dot{x}_1 &= -(r_s / L_{ss})\, x_1 - (\psi_m / L_{ss})\, x_3 \cos x_4 + (u_{as} + \Delta u_{as}) / L_{ss} \\ \dot{x}_2 &= -(r_s / L_{ss})\, x_2 + (\psi_m / L_{ss})\, x_3 \sin x_4 + (u_{bs} + \Delta u_{bs}) / L_{ss} \\ \dot{x}_3 &= (P\psi_m / 2J)\, x_1 \cos x_4 + (P\psi_m / 2J)\, x_2 \sin x_4 - (B_m / J)\, x_3 - T_L / J - \Delta T_L / J \\ \dot{x}_4 &= x_3 \end{aligned} \qquad (17)$$

$$y_1 = x_1 + \Delta i_{as}$$
$$y_2 = x_2 + \Delta i_{bs}$$

To apply the EKF and UKF to the motor, it is necessary to define the states of the system in matrix form. The state vector x and the measurement vector y can be defined, respectively, as

$$x = [\, i_{as} \quad i_{bs} \quad \omega_{rm} \quad \theta_{rm} \,]^T, \quad y = [\, i_{as} \quad i_{bs} \,]^T.$$

Then, the system equation can be described by

$$\left.\begin{aligned} \dot{x} &= f(x,u) + B_L\; T_L + \mathbf{w} \\ y &= h\, x + \mathbf{v} \end{aligned}\right\} \qquad (18)$$

where

$$f(x,u)=\begin{bmatrix}(-r_s/L_{ss})\ x_1-(\psi_m/L_{ss})\ x_3\ \cos x_4+u_{as}/L_{ss}\\ (-r_s/L_{ss})\ x_2+(\psi_m/L_{ss})\ x_3\ \sin x_4+u_{bs}/L_{ss}\\ (\psi_m/2J)\ x_1\ \cos x_4+(3\psi_m/2J)\ x_2\ \sin x_4-(B_m/J)\ x_3\\ x_3\end{bmatrix}$$

$$B_L=\begin{bmatrix}0 & 0 & -T_L/J & 0\end{bmatrix}^T$$

the process noise vector $\boldsymbol{w}$ and measurement noise vector $\boldsymbol{v}$ are given by

$$\boldsymbol{w}=\begin{bmatrix}\Delta u_{as}/L_{ss} & \Delta u_{bs}/L_{ss} & -\Delta T_L/J & 0\end{bmatrix}^T,$$

$$\boldsymbol{v}=\begin{bmatrix}\Delta u_{as} & \Delta u_{bs}\end{bmatrix}^T$$

If a discrete EKF is used and the data fed to UKF is in discrete form, then a discretizing form of system model of Eq.(17) would be required. The result of discretization gives the discrete version of Eq.(3) [7],

$$\left.\begin{aligned}x_{k+1}&=f(x_k,u_k)+w_k\\ y_k&=h(x_k)+v_k\end{aligned}\right\}\qquad (19)$$

where

$$f(x_k,u_k)=\begin{bmatrix}x_{1k}\\ x_{2k}\\ x_{3k}\\ x_{4k}\end{bmatrix}+\begin{bmatrix}f_1(x_k,u_k)\\ f_2(x_k,u_k)\\ f_3(x_k,u_k)\\ f_4(x_k,u_k)\end{bmatrix}T,$$

$$f_1(x_k,u_k)=-(r_s/L_{ss})\ x_{1k}+(\psi_m/L_{ss})\ x_{3k}\ \cos x_{4k}+u_{ask}/L_{ss}$$

$$f_2(x_k,u_k)=-(r_s/L_{ss})\ x_{2k}+(\psi_m/L_{ss})\ x_{3k}\ \cos x_{4k}+u_{bsk}/L_{ss}$$

$$f_3(x_k,u_k)=(P\psi_m/2J)\ x_{1k}\ \cos x_{4k}+(P\psi_m/2J)\ x_{2k}\sin x_{4k}-(B_m/J)x_{3k}$$

$$f_4(x_k,u_k)=x_{3k}$$

$$h(x_k,u_k)=\begin{bmatrix}x_{1k} & x_{2k}\end{bmatrix}^T,$$

$$\boldsymbol{v}_k=\begin{bmatrix}\Delta u_{ask} & \Delta u_{bsk}\end{bmatrix}^T,$$

$$\boldsymbol{w}_k=\begin{bmatrix}\Delta u_{ask}/L_{ss} & \Delta u_{bsk}/L_{ss} & -\Delta T_L/J & 0\end{bmatrix}^T$$

where T is the step size and the superscript T indicates a matrix transpose.

Let's also suppose that it is possible to measure the motor winding currents, and we want to use the EKF and UKF to estimate machine states. The connection of estimator (EKF or UKF) with motor is shown in Figure (3).

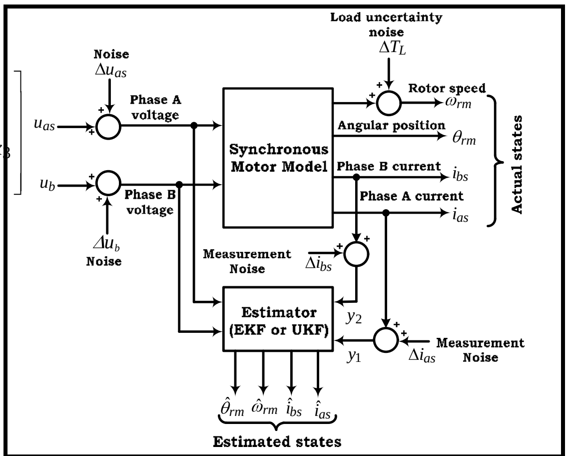

Figure (3) The connection of the estimator with motor model

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To estimate motor states, the estimator has to receive noise-corrupted phase voltages ($u_{as}+\Delta u_{as}$) and ($u_{bs}+\Delta u_{bs}$), and, also, it should measure (noisy) phase currents, y_1 and y_2. Then based on special algorithms, the estimator would estimate the states of phases currents ($\hat{i}_{as}, \hat{i}_{bs}$), rotor speed $\hat{\omega}_{rm}$ and rotor angular position $\hat{\theta}_{rm}$.

Scalar Control of PMSM

Constant volt per hertz control in an open loop control represents the most common control strategy for asynchronous motors. Using this technique for synchronous motors with permanent magnets offers a big advantage of sensorless control [8,9]. To maintain the stator flux constant at its nominal value in the base speed range, the voltage-to-frequency ratio is kept constant, hence the name V/f control. If the ratio is different from the nominal one, the motor will become overexcited or under-excited. The first case happens when the frequency value is lower than the nominal one and the voltage is kept constant or if the voltage is higher than that of the constant ratio V/f. The over-excitation condition means that the magnetizing flux is higher than its nominal value. An increase of the magnetizing flux leads to a rise of the magnetizing current. In this case the hysteresis and eddy current losses are not negligible. The second case represents under-excitation. The motor becomes under-excited because the voltage is kept constant and the value of stator frequency is higher than the nominal one [8,9]. Such a control strategy can be represented by the block diagram illustrated in Fig.(4). The Kalman estimators have been inncluded to estimate the motor states. As shown in the figure, the estimators need for their work a direct measurement of voltages at the output of voltage source inverter. The estimators, also, requires sensing of current through the stator phases.

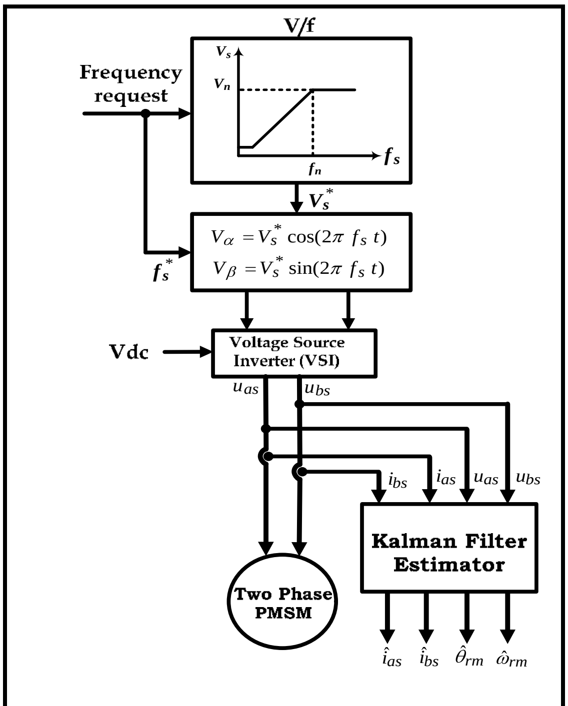

Figure (4) Block diagram of Volt per Hertz control of PMSM

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EKF Algorithm for Motor State Estimation:

In any Kalman-based filter, both a model of the process and a model measurement are required,

$$x_{k+1} = f(x_k, u_k) + w_k \qquad (20)$$

$$y_k = h(x_k) + v_k$$

where w_k is the process noise and v_k is the measurement noise. x_k is called the state of the system. u_k is a known input to the system (called the control signal) and y_k is the measured output.

If either the process or measurement equation is nonlinear, this violates the linear assumption of the standard Kalman filter. The extended Kalman filter (EKF) is an ad hoc technique to provide to use the standard Kalman filter on non-linear process or measurement models resulting in sub-optimal estimates. The measurement model and process model are linearized about the mean and covariance (the current operating point) at each iteration and the standard Kalman filter is applied to the linearized models. The linearization has been approximated in the extended Kalman filter using a first order Taylor expansion. To accomplish this, the Jacobian matrix of both the process model and the measurement model need to be calculated [1, 2, 10].

In case of two-phase PMSM, one can easily deduce from Eq.(18) that the process equation is nonlinear and the measurement is linear. Therefore, calculation of Jacobian matrix for measurement is trivial, while for process is nontrivial. In order to use an EKF, one need to find the derivatives of $f(x_k, u_k)$ and $h(x_k)$ with respect to x_k at each time step and evaluated at the current state estimate, i.e

$$A_k = f'(\hat{x}_k, u_k) = \frac{\partial f(\hat{x}_k, u_k)}{\partial x} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix},$$

$$C_k = h'(\hat{x}_k) = \frac{\partial h(\hat{x}_k)}{\partial x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where

$a_{11} = -r_s / L_{ss}$, $a_{12} = 0$, $a_{13} = -(\psi_m / L_{ss}) \cos \hat{x}_{4k}$,

$a_{14} = (\psi_m / L_{ss})\, \hat{x}_{3k} \sin \hat{x}_{4k}$, $a_{21} = 0$, $a_{22} = -r_s / L_{ss}$

$a_{23} = (\psi_m / L_{ss}) \sin \hat{x}_{4k}$, $a_{24} = (\psi_m / L_{ss})\, \hat{x}_{3k} \cos \hat{x}_{4k}$

$a_{31} = (P \psi_m / 2J) \cos \hat{x}_{4k}$, $a_{32} = (P \psi_m / 2J) \sin \hat{x}_{4k}$,

$a_{34} = (P \psi_m / 2J)[\hat{x}_{2k} \cos \hat{x}_{4k} - \hat{x}_{1k} \sin \hat{x}_{4k}]$,

$a_{33} = -B_m / J$

$a_{41} = 0$, $a_{42} = 0$, $a_{43} = 1$, $a_{44} = 0$

After linearizing the nonlinear model of synchronous motor, one can execute the following the standard Kalman filter equations [1, 2, 10]:

$$K_k = P_k C_k^T \left(C_k P_k C_k^T + R_k \right)^{-1}$$

$$\hat{x}_{k+1} = f(\hat{x}_k, u_k) + K_k \left[y_k - h(\hat{x}_k) \right] \qquad (21)$$

$$P_{k+1} = A_k (I - K_k C_k) P_k A_k^T + Q_k$$

where $\hat{x}_k$ is the estimate of x_k, K_k is called the Kalman gain, Q_k is the covariance of the process noise (w_k) and R_k is the covariance of the measurement noise (v_k).

The unscented Transformation

The problem of predicting the future state or observation of the system can be expressed in the following form. Suppose that $\boldsymbol{x}$ is n-dimensional vector random variable with mean $\bar{\boldsymbol{x}}$ and covariance $\boldsymbol{P_{xx}}$. A second m-dimensional random vector variable $\boldsymbol{y}$ is related to $\boldsymbol{x}$ through the nonlinear transformation $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$. One would like to calculate the mean $\bar{\boldsymbol{y}}$ and covariance $\boldsymbol{P_{yy}}$ of y [3, 11].

The unscented transformation is a new, novel method for calculating the statistics of a random variable which undergoes a nonlinear transformation. It is found on the intuition that it is easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function of transformation. The approach is illustrated in Fig.(5). A set of points (or sigma points) are chosen so that their sample mean and sample covariance are $\bar{x}$ and P_{xx}. The nonlinear function is applied to each point in turn to yield a cloud of transformed points and $\bar{y}$ and P_{yy} are the statistics of the transformed points.

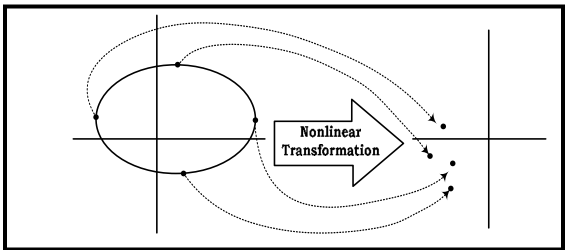

Figure (5) The principle of the unscented transformation

Given an n-dimensional Gauassian distribution having covariance P, one can generate a set of $O(n)$ points having the same sample covariance from the columns (or rows) of the matrices $\pm\sqrt{nP}$ (the positive and negative roots). This sets of points is zero mean, but if the original distribution has mean $\bar{x}$ to each of the points yields a symmetric set of 2n points having the desired mean and covariance. Because the set is symmetric its odd central moments are zero, so its first three moments are the same as the original Gaussian distribution. The transformation procedure can be summarized as [3,11]:

1. Compute the set σ of $2n$ points from the rows or columns of the matrices $\pm\sqrt{(n+\kappa)P_{xx}}$. This set is zero mean with covariance P_{xx}.

   $\sigma \leftarrow$ $2n$ rows or columns from $\pm\sqrt{(n+\kappa)P_{xx}}$

where $\kappa \in \Re$.

2. Compute a set of points with the same covariance, but with mean $\bar{x}$, by translating each of the points as

$\chi_o = \bar{x}$	22

$\chi_i = \bar{x} \pm \left(\sqrt{(n+\kappa)P_{xx}}\right)_i$	23

which assures that

$P_{xx} = \sum_{i=1}^{2n} W_i^c [\chi_i - \bar{x}][\chi_i - \bar{x}]^T$	24

The transformed set of sigma points are evaluated for each of the 0-2n points by

$y_i = f(\chi_i)$	25

3. The predicted mean and covariance are computed as

$\bar{y} = \left\{ W_o^m y_o + \sum_{i=1}^{2n} W_i^m y_i \right\}$	26

$P_{yy} = \left\{ W_o^c [y_o - \bar{y}][y_o - \bar{y}]^T + \sum_{i=1}^{2n} W_i^c [y_i - \bar{y}][y_i - \bar{y}]^T \right\}$	27

where $y_o = f(\chi_o)$. Each of the sigma vectors is assigned with a weight. These weights are calculated by the following equations:

$W_o^m = W_o^c = \kappa/(n+\kappa)$	28

$W_i^m = W_i^c = 1/2(n+\kappa) \quad i = 1,\ldots,2n$	29

In the present work, the value of κ is set to zero.

The unscented Kalman Filter:

For nonlinear systems, the hybrid extended Kalman filter (EKF) is considered to be the best nonlinear estimator. However, as discussed previously, the EKF has some limitations as it is based on the linearization of the nonlinear system and also on some other approximations. The unscented Kalman filter is an alternative to the EKF which has the implementation of unscented transformation of the nonlinear state distribution and then applying the recursive Kalman filter algorithm for the time update and measurement update for the nonlinearly transformed state distribution [4,11].

The unscented Kalman filter algorithm can be divided in to three sections. The first part is the initialization of the state estimate and state covariance of the nonlinear system. The second part is applying the UT to the state distribution and calculating the a priori state estimate and a priori state covariance. The third part is performing the measurement update equations and calculating the Kalman gain, state estimate and state error covariance.

Let us again consider the discrete time nonlinear system of the motor model represented by Eq.(19) and given by:

$$x_{k+1} = f(x_k, u_k, t_k) + w_k$$
$$y_k = h(x_k, t_k) + v_k$$

where w_k and v_k are additive process and measurement noise, with zero mean and covariances of Q_k and R_k . The unscented Kalman filter algorithm can be listed as follows [4, 11-14]:

1. Initialization

The UKF is initialized with the initial estimate and estimation error covariance as in the EKF.

$$\hat{x}_0^+ = E(x_0) \quad (30)$$

$$P_0^+ = E[(x_o - \hat{x}_o^+)(x_o - \hat{x}_o^+)^T] \quad (31)$$

where E(.) denotes expectation function, $\hat{x}_0^+$ and P_0^+ denotes posteriori of estimate and its corresponding covariance matrix, respectively.

2. Sigma point selection:

As seen previously in the unscented transformation, a set of sigma points and their corresponding weights are calculated around the initial estimate according to Eq.(22) and (23)

$$\hat{x}_{k-1}^i = \hat{x}_{k-1}^+ \qquad i = 0; \quad (32)$$

$$\hat{x}_{k-1}^i = \hat{x}_{k-1}^+ + \left(\sqrt{nP_{k-1}^+}\right)_i \qquad i = 1,\ldots,n \quad (33)$$

$$\hat{x}_{k-1}^i = \hat{x}_{k-1}^+ - \left(\sqrt{nP_{k-1}^+}\right)_i \qquad i = n+1,\ldots,2n \quad (34)$$

3. Time Update

The system gets updated from k-1 to k time step. All the sigma points $\hat{x}_{k-1}^i$ are propagated through the nonlinear function *f(.)* and *h(.)* and then the corresponding nonlinear sigma points $\hat{x}_k^i$ are obtained.

$$\hat{x}_k^i = f(\hat{x}_{k-1}^i, u_k, t_k) \quad (35)$$

$$\hat{y}_k^i = h(\hat{x}_k^i, t_k) \quad (36)$$

Using the $\hat{x}_k^i$ vectors and also the weights W_i^c and W_i^m , one has to perform the following steps.

(a) The a priori state estimate $\hat{x}_k^-$ ^ at time t_k is calculated as

$$\hat{x}_k^- = \sum_{i=0}^{2n} W_i^m \ \hat{x}_k^i \quad (37)$$

b) The a priori estimation error covariance is calculated. However, one should add Q_{k-1} to the end of the equation to take the process noise into account:

$$P_k^- = \sum_{i=0}^{2n} W_i^c (\hat{x}_k^i - \hat{x}_k^-)(\hat{x}_k^i - \hat{x}_k^-)^T + Q_{k-1} \quad (38)$$

Similarly using the $\hat{y}_k^i$ vectors (measurements from sigma points) $\hat{y}_k^-$ is calculated as

$$\hat{y}_k^- = \sum_{i=0}^{2n} W_i^m \ \hat{y}_k^i \quad (39)$$

4. Measurement Update

Using the calculated a priori state estimate, a priori estimation error covariance and measurement estimate, the following terms are calculated.

a) Computation the covariance of the predicted measurement

$$P_{yy} = \sum_{i=0}^{2n} W_i^c (\hat{y}_k^i - \hat{y}_k^-)(\hat{y}_k^i - \hat{y}_k^-)^T + R_k \quad (40)$$

b) Estimation the cross covariance between $\hat{x}_k^-$ and $\hat{y}_k^-$ as

$$P_{xy} = \sum_{i=0}^{2n} W_i^c (\hat{x}_k^i - \hat{x}_k^-)(\hat{y}_k^i - \hat{y}_k^-)^T \quad (41)$$

c) The measurement update of the state estimate and estimate error covariance is performed using the general Kalman filter equations by calculating the Kalman gain K_k

$$K_k = P_{xy} P_{yy}^{-1}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - \hat{y}_k^-) \quad (42)$$

$$P_k^+ = P_k^- - K_k \, P_{yy} \, K_k^T$$

Figure (6) shows the flow chart which summarizes the UKF algorithm. It is worth to mention that the flow chart is based on setting the scaling parameter κ equal to zero.

Results

The state estimation process of two-phase PMSM has been modeled and implemented using Simulink shown in Fig.(7). To account for machine parameter variations, the machine model has been coded in m-file and added to the Simulink using S-function block. Moreover, to implement an online estimation, the blocks of EKF and UKF estimators are also added to Simulink models using the S-function capability. S-functions use a special calling method that enables users to interact with Simulink equation solvers. The algorithms of the estimators and the model are coded in m-files with the same names as their corresponding S-function blocks. During simulation of a model, at each simulation stage, Simulink calls the m-files of process and estimators and, also, it calls the appropriate methods for each S-function block in the model and then it would yield the outputs of S-function blocks immediately after each sampling instant. The form of an S-function can accommodate continuous and discrete systems.

The V/F strategy has been implemented in Simulink portrait of Fig.(7) using a look-up table block. The look-up table holds the proportionality relationship between the frequency and the phase voltage amplitude to give constant flux operation. The simulation time base is combined with the required phase voltage amplitude and frequency to give two balanced phase voltages.

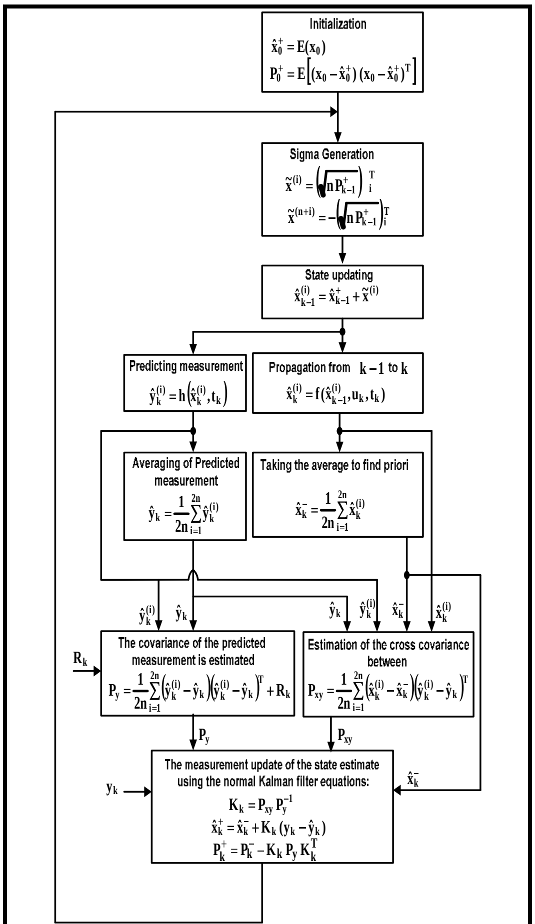

Figure (6) Flow chart of performing UKF algorithm

As indicated in the Fig.(7), the S-function block of the machine model receive the quadrature phase voltages and exerted load to yield the true estimates of phase currents, rotor angular speed and position. On the other hand, the EKF and UKF estimators take the stator phase voltages at machine input and the current measurements from the S-function block output of the machine model and give the estimates of the machine variables.

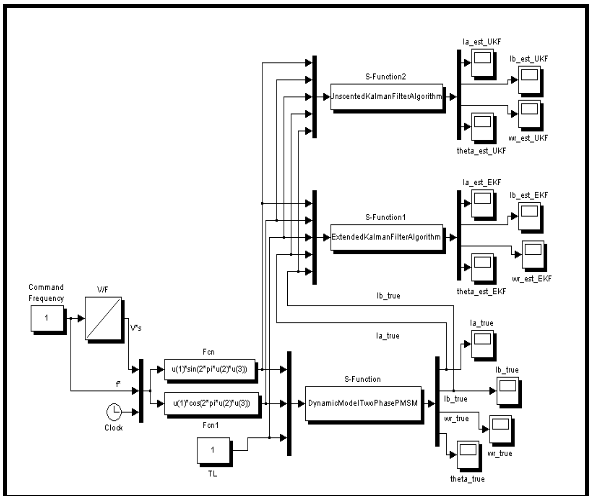

Figure (7) SIMULINK Modeling of Motor State Estimation System

The noise contamination of measurements and states has been simulated inside m-files of estimators' algorithms. It will be assumed that the state, measurement and load uncertainty noises are white noises with zero-mean. Their standard deviations have been assigned in Table (1). Table (1) also lists the values of parameters and coefficients of the system. The initial conditions of the system states and the error covariance matrix are given as

$$x_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \hat{x}_0^+ = x_0 \quad P_0^+ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since the estimation process deals with a discrete form of estimators, a fixed-step type with 2ms has been selected in the simulation parameters and 2 seconds stop-time has been adopted for most simulations.

Table (1) System coefficients and parameters	
Parameter	**value**
Winding resistance (R_s)	1.9 Ω
Winding inductance (L_{ss})	0.003 H
Flux constant of motor (ψ_m)	0.1 Weber
Moment of inertia (J)	0.00018 N.m. s^2
Coefficient of viscous friction (F)	0.001 N.m.s
Input frequency *(f)*	1 Hz
Standard deviation of measurement noises (Δi_{as} , Δi_{bs})	0.1 Amp
Standard deviation of phase voltage noises (Δu_{as} , Δu_{bs})	0.001 volt
Standard deviation of noise due to torque disturbance (ΔT_L)	0.05 $rad/\sec^2$

Figures (8)-(10) show the true and estimated states (winding currents, rotor angular velocity and position of synchronous machine) when the machine is operated at 1 Hz source frequency. One can easily see that both the EKF and UKF could estimate all the states of the motor. It is clear that the estimates resulting from the UKF estimator are closer to the true states than those obtained from EKF.

The performance of both estimators can be assessed via estimation error portraits. Figure (11) shows the standard deviation of state estimation errors (for all states) obtained from both filters and at rotor speed of 6.2832 rad/sec. The average RMS estimation errors of the EKF and UKF (six sigma points since we chose *W(0)* = 0), are calculated and listed in Table (2). It is clear from the figures and Table (2) that the UKF well-performs for estimation of all states (winding currents, velocity and angular position). It is seen from Table (2) that the UKF consistently gives estimates that are one or two orders of magnitude better than the EKF.

Table (2) Average of RMS of the state estimation Errors

State	EKF	UKF
Widing A Current (A)	1.3313	0.2418
Widing B Current (A)	1.4901	0.2726
Rotor speed (rad/s)	23.1698	5.5201
Rotor position (rad)	2.7265	0.6492

It is interesting to examine the performance of both estimator at different frequency and to check if the UKF could keep its superiority over a wide range of speed and frequency. The new suggested source frequency has been chosen to be 10 times the previous one. Figure (12) shows the RMS value of estimation errors for different states with this new frequency. It is evident from the figure that the UKF degrade at this frequency and the EKF shows better estimation performance than UKF. However, at this frequency the EKF shows good characteristics in terms of rotor angular speed and position. The performances of both estimators, in case of current state, are evenly equal as shown in the figure.

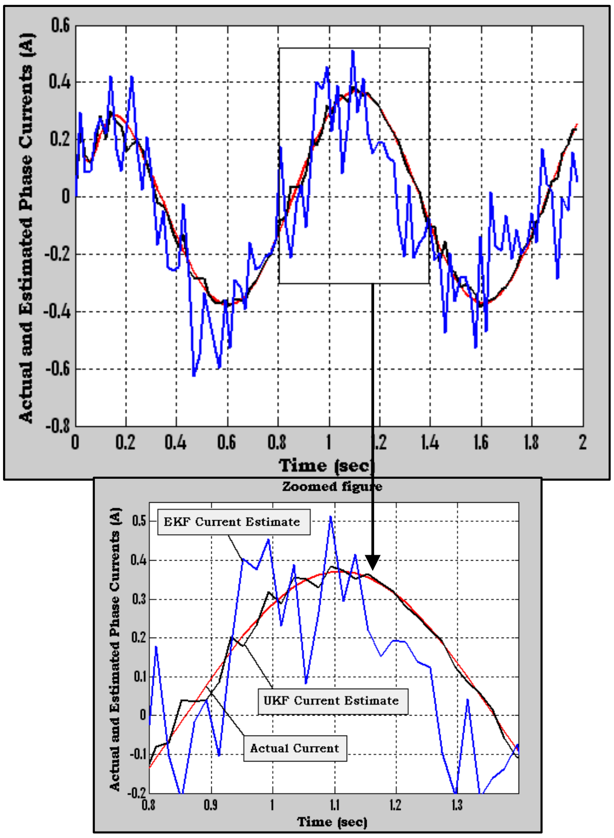

Figure (8) True and estimated states of phase current (at source frequency 1Hz).

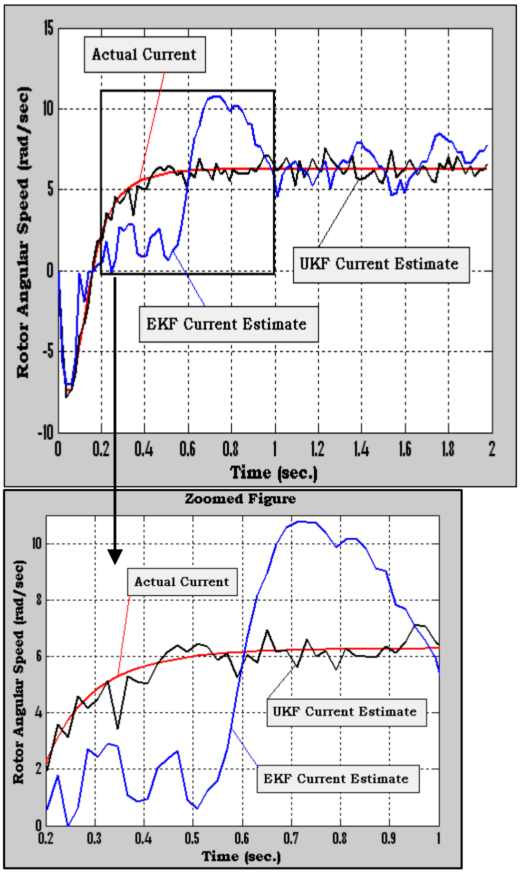

Figure (9) True and estimated states of rotor speed
(at source frequency 1Hz.)

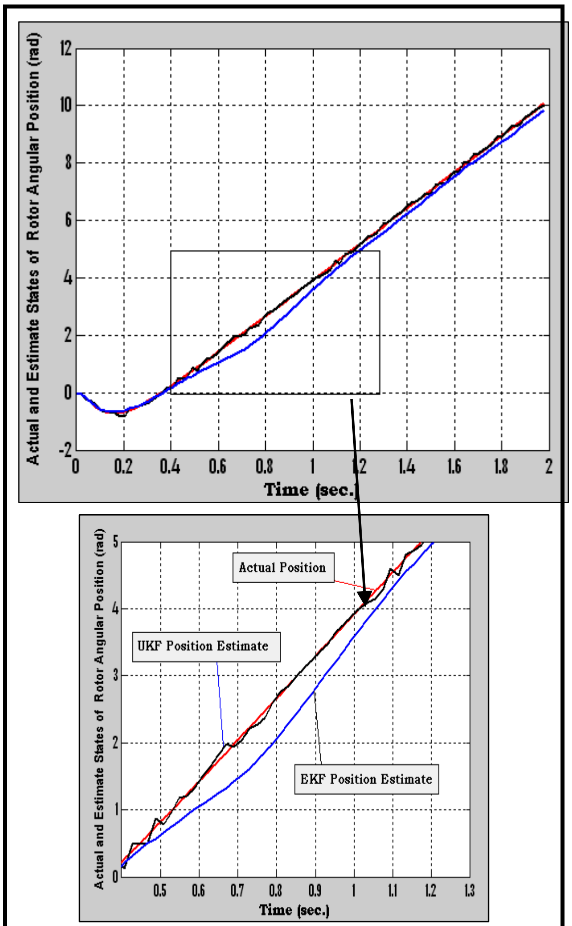

Figure (10) True and estimated states of rotor angular position (at source frequency 1Hz.)

The actual and estimate states of rotor speed variable have been simulated at rotor speed of 10 Hz source frequency (62.832 rad/sec.) as shown in Fig.(13) . It is evident from the figure that the speed response estimate due to EKF is closer to actual state than the corresponding estimate obtained from EKF However, the average of RMS value of the estimation error resulting from both filters for each state and over source frequency range 1-10Hz has been calculated and illustrated in Fig.s (14)-(16). One can see that the assessment of both filter performances depends on the state and the value of fed frequency. For the case of rotor speed state, the performance of UKF improves at low frequency and degrades at high frequency, while the performance of EKF degrade at low frequency and then it shows constant characteristics at higher frequency. On the other hand, for the case of current state, the average of RMS value resulting from EKF is higher than that obtained from UKF at low frequency. The averages of estimation errors generated from both estimators are approximated equal at frequency of 10 Hz (62.832 rad/sec.). This result is evident in Fig.(12), where the average of RMS value of the current estimation error is approximately equal. Therefore, one can conclude that the performance of UKF outperforms the EKF at low frequency and gives bad estimation characteristics at high frequency. Meanwhile, the EKF estimator generates bad estimates at low frequency but it keeps its characteristics and shows better performance at higher frequency.

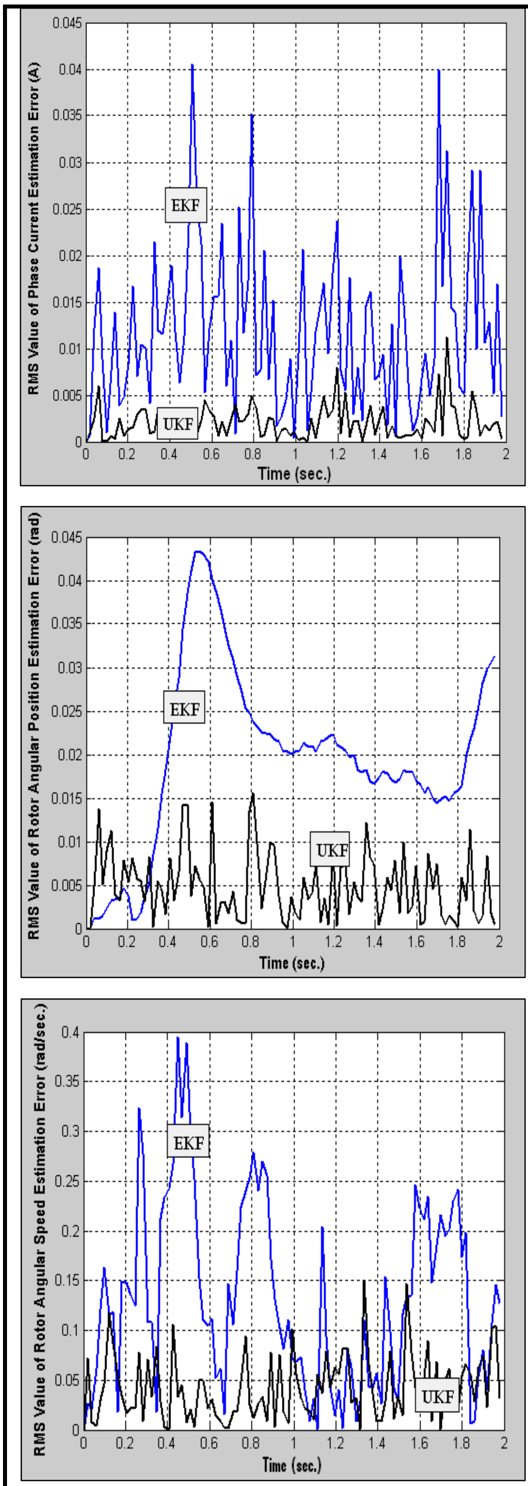

Figure (11) RMS values of different state estimation errors
(at source frequency 1Hz.)

The **P** matrix quantifies the uncertainty in the state estimates. In other words, the **P** matrix give us an idea or an indication of how accurate our estimates are. Figure (17) gives the behavior of the sum of diagonal elements (trace) of matrix P for both EK and UK filters at source frequency 1 Hz. The figure shows that the UKF has more confidant with its estimates than that the EKF has. This is evident from the difference of magnitudes between covariance matrices in both filters. The high values of **P** in EKF gives an indication that its estimates is of low certainty and then with large errors. However, the trace of **P** matrix with both filters would later lower and the confidence of producing an accurate estimate would rise.

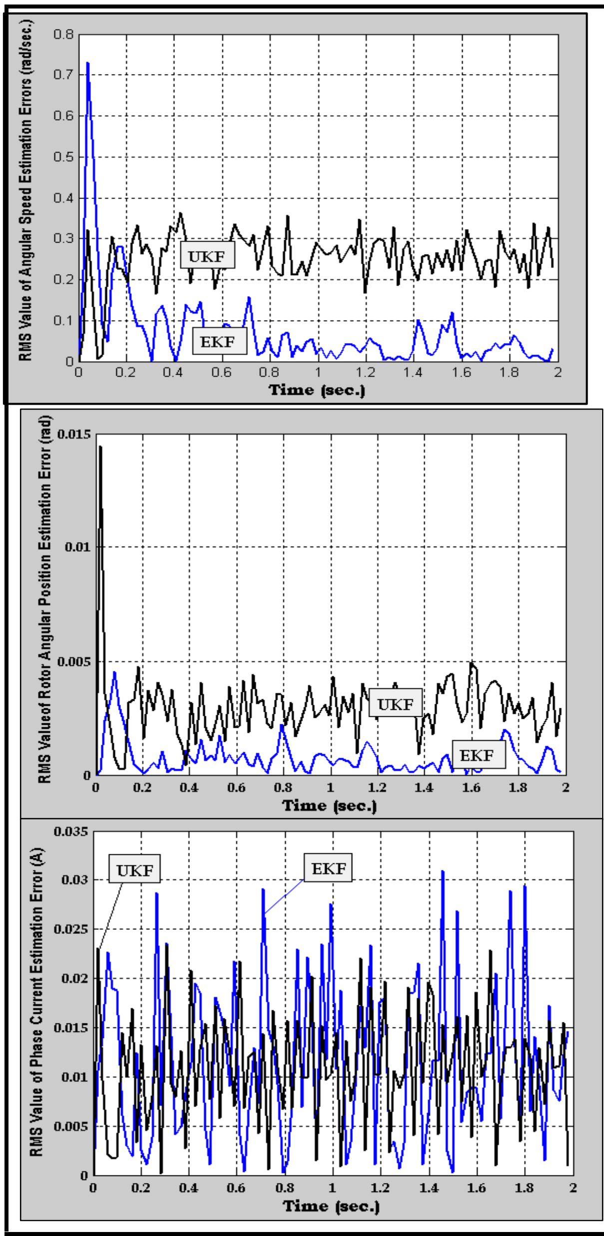

Figure (12) RMS values of different state estimation errors
(at source frequency 1Hz.)

It is of importance to assess the performance of both estimators in terms of computation effort of their software algorithms. The Matlab functions "tic" and "toc" work together to measure elapsed time. The sequence of these commands can be employed to measure the amount of time the MATLAB software that takes to complete one or more operations and displays the time in seconds. The calculation effort of EKF and UKF can be assessed using Fig.(18). At each program iteration, the effective time required to calculate the steps of each filter algorithm is computed. It is clear from Fig.(19) that the average time required to execute the UKF algorithm over all the simulation time is higher than that with EKF algorithm. However, the simulation is implemented with 6.2832 (1 Hz) rotor angular speed. It is necessary to see the execution time taken by both filters at source frequency of 10 Hz or at rotor speed of 62.832 rd/sec. The result shown in Fig.(18) assures that the EKF still has a lower execution time than its counterpart.

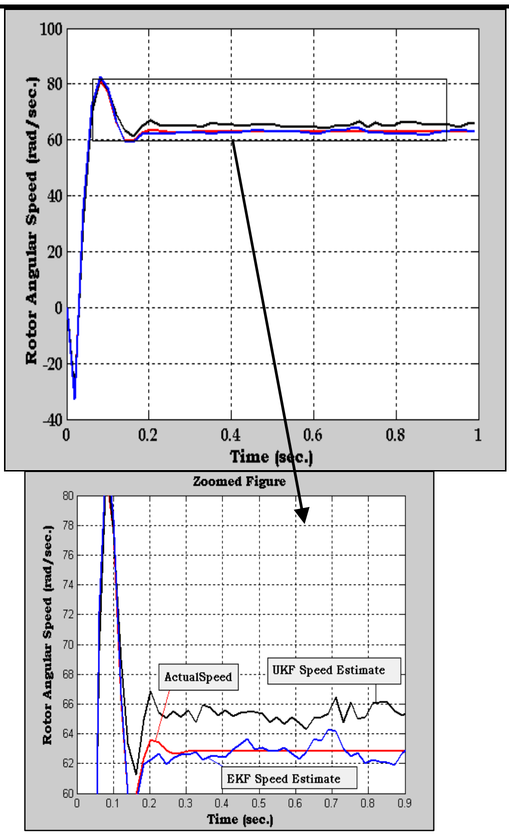

***Figure* (13)** RMS values of different state estimation errors
(at source frequency 1Hz.)

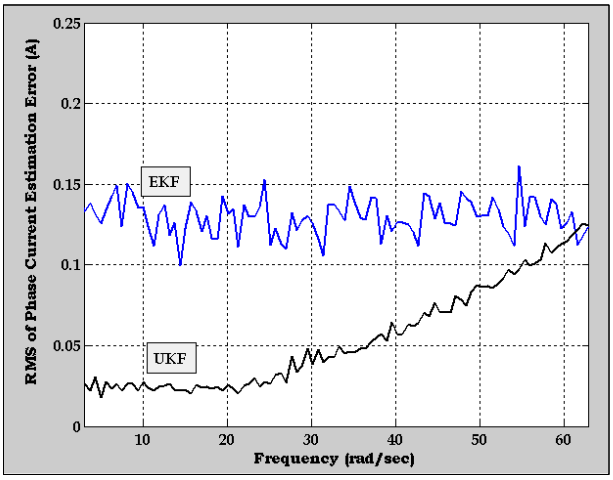

Figure (14) Mean of RMS value of the current estimation error over a wide range of frequencies (1-10Hz.)

In Fig.(20), the average of execution time over simulation run has been calculated at each rotor angular speed up to 62.832 rd/sec . One can conclude from the figure that EKF always has a lower execution time over the prescribed range of speed than that time required to execute the UKF algorithm.

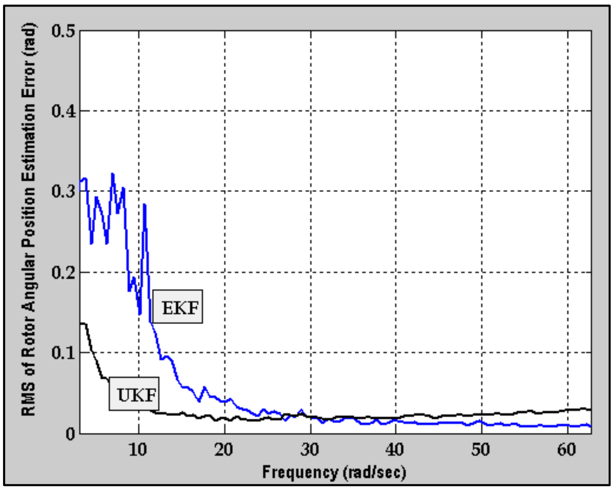

Figure (15) Mean of RMS value of the current estimation error over a wide range of frequencies (1-10Hz.)

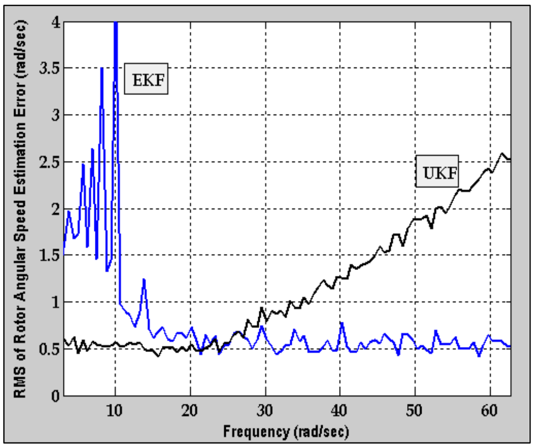

Figure (16) Mean of RMS value of the current estimation error over a wide range of frequencies (1-10Hz.)

Conclusion:

- The simulated results shows that the unscented filter can give greatly improved estimation performance compared with the extended Kalman filter at low rotor speed. However, its performance would degrade gradually as the rotor speed has been increased.

- On the other hand, the EKF gives bad state estimates at low rotor speed. However it keeps its estimation characteristics and yield better performance than its opponent at higher speed.

- Results showed that UKF has lower values of covariance matrix trace than that with EKF. This gives an indication that UKF is more confidant with its estimates than the EKF.

- The EKF requires the computation of Jacobians (partial derivative matrices), while the UKF does not use Jacobians. For systems with analytic process and measurement equtions, it is easy to compute Jacobians. But some systems are not given in analytical form and it is numerically difficult to compute Jacobians.

- For the considered system specifically, the average execution time required to calculate the UKF algorithm (for each iteration) is higher than that required to calculate EKF algorithm.

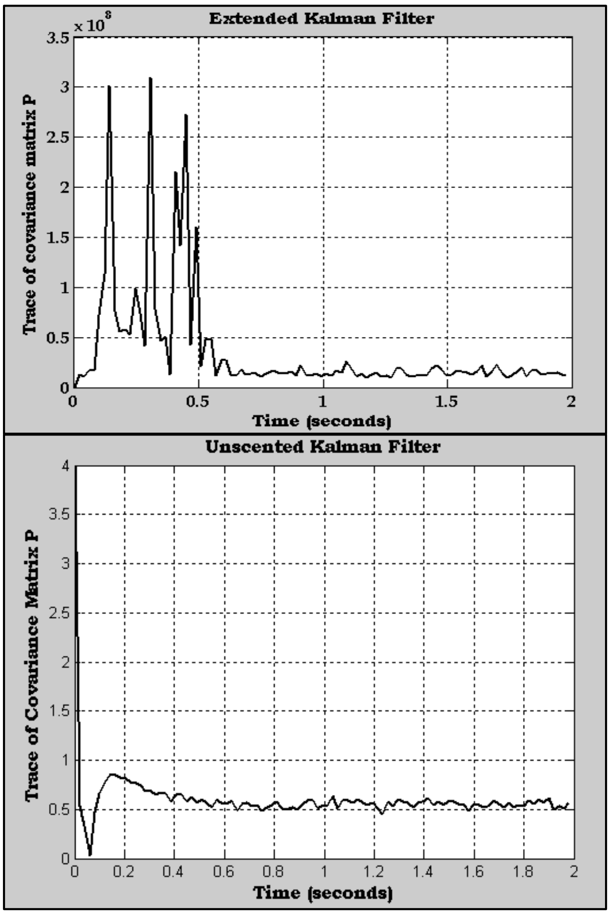

Figure (17) Behavior of covariance matrices **P** of both EKF and UKF with 1Hz source frequency.

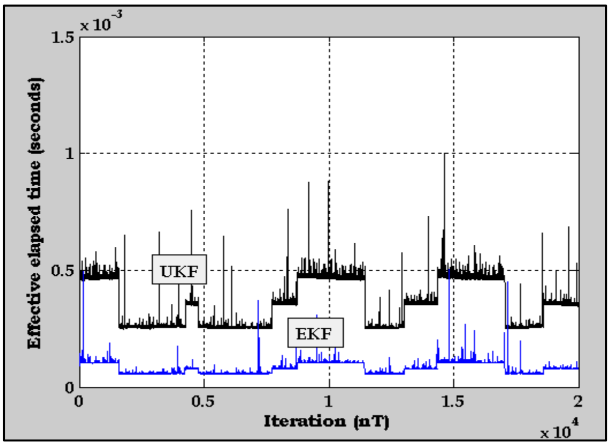

Figure (18) Calculation effort for both estimators at 1Hz source frequency

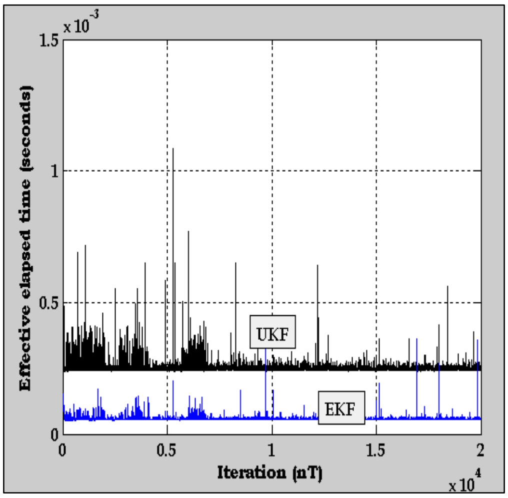

Figure (19) Calculation effort for both estimators at 10 Hz source frequency

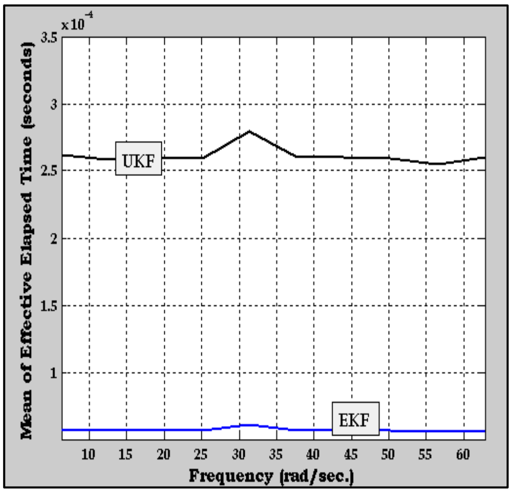

Figure (20) Mean of execution time over source frequency range 1-10Hz

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مقارنة اداء مخمنين لمحرك تزامني ذو طورين وذات مغانيط دائمية

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الخلاصة:

يقدم البحث نموذجين من مرشحات كالمان (Kalman Filters)؛ وهما مرشح كالمان المعدل (Extended Kalman Filter) ؛ و(unscented Kalman filter) وذلك لتخمين متغيرات المنظومة (states) وتشمل تيارات ملفات المجال، وزاوية والسرعة الزاوية للجزء الدوار (rotor). يعتمد مرشح كالمان المعدل في تخمين المتغيرات على تحويل ألانموذج اللاخطي الى أنموذج خطي. لذلك من الضروري حساب مصفوفة جاكوبين (Jaccobian Matrix) في كل لحظة زمنية. بينما يعتمد (unscented Kalman filter) على توليد عينات من النقاط بطريقة معينة يمكن استخدامها لغرض حساب المتوسط الحسابي (mean) والتباين (covariance) للمتغيرالعشوائي ذو توزيع (Gaussian)، حيث يتم تمرير هذه النقاط بالانموذج اللاخطي للمنظومة وايجاد، مرة اخرى، المتوسط الحسابي (mean) والتباين (covariance) لناتج خرج هذه النقاط لدرجة دقة تصل الى حدين فقط من متوالية تيلر (Taylor series). اثبت النتائج ان (UKF) يتميز باداء عالي للتخمين في حالة السرع الواطئة للجزء الدوار، بينما يتفوق اداء (EKF) في حالة السرع العالية للمحرك التزامني. وكذلك فان الزمن المستغرق لتنفيذ خوارزمية (EKF) هي اقل من الزمن المطلوب لتنفيذ خوارزمية (UKF) ولجميع مديات السرع للمحرك التزامني