Analytical Solution For Buckling Of Laminated Conical Shells

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Abstract:

Buckling analysis of laminated conical shells under axial compressive load are investigated analytically using high order and Love's shell theories. Power series are used to solve the developed equations of motion for conical shells with different semi vertex angles, length- to – radius ratio, number of layers and boundary conditions. The validity of the presented procedure is confirmed.

Keywords:

Buckling, conical, composite, laminate, shel

Introduction:

Due to their extensive use, particularly in aeronautical industry, the buckling of conical shells has been studied by many researchers, **(SOFIYEV Abdullah H. 2003), s**tudied the buckling of an orthotropic composite truncated conical shell with continuously varying thickness, and subjected to a uniform external pressure which is a power function of time. At first, the fundamental relations and the Donnell type stability equations of an orthotropic composite truncated conical shell, subjected to an external pressure, have been obtained. Then, by employing Galerkin method, those equations have been reduced of time dependent differential equation with variable coefficients. Finally, by applying the variational method of Ritz method type, the critical static and dynamic loads, the corresponding wave numbers and the dynamic factor have been found analytically.

(A. H. Sofiyev and O. Aksogan 2004), considered the buckling of an elastic truncated conical shell having a meridional thickness expressed by an arbitrary function, subject to a uniform external pressure, which is a power function of time. At first, the fundamental relations and Donnell type dynamic buckling equation of an elastic conical shell with variable thickness have been obtained. Then, employing Galerkin's method, those equations have been reduced to a time-dependent differential equation with variable coefficients. Finally, applying the Ritz type variational method, the critical static and dynamic loads, the corresponding wave numbers, dynamic factor and critical stress impulse have been found analytically.

(SHKUTIN L. I. 2004), studied the nonlinear boundary value problem of the axisymmetric buckling of a simply supported conical shell (dome) under a radial compressive load applied to the supported edge, and formulate a system of six first order ordinary differential equations for independent fields of finite displacements and rotations.

(Rajesh K. Bhangale et al 2006), obtained multi valued solutions using the shooting method with specified accuracy. A finite element formulation based on. First-Order Shear Deformation Theory (FSDT) is used to study the thermal buckling and vibration behavior of truncated FGM conical shells in a high-temperature environment. A Fourier series expansion for the displacement variable in the circumferential direction is used to model the FGM conical shell. The material properties of the truncated FGM conical shells are functionally graded in the thickness direction according to a volume fraction power law distribution. Temperature dependent material properties are considered to carry out a linear thermal buckling and free vibration analysis. The conical shell is assumed to be clamped–clamped and has a high temperature specified on the inner surface while the outer surface is at ambient temperature. The one-dimensional heat conduction equation is used across the thickness of the conical shell to determine the temperature distribution and thereby the material properties. In addition, the influence of initial stresses on the frequency behavior of FGM shells has also been investigated.

(Francesca Guana and Franco Pastrone 2007), dealed with the problem of equilibrium and buckling of nonlinear elastic axisymmetric shells, whose referential shape is a truncated circular cone, subject to compressive end loadings. They considered thin Kirchhoff shells and proved, by means of the bifurcation theory of Poincaré, the non-uniqueness of solutions of the boundary value problem associated with the equilibrium equations, the assigned end loadings and the geometrical constraints: i.e., the axisymmetry and the inextensibility along meridians. The critical loads are determined as well as the bifurcation points. If the material is hyper elastic the equations of equilibrium are derived from a variational principle and, for some special form of the strain energy function, a Hamiltonian formulation can be provided. The possibility of a non-convex strain energy function is briefly discussed.

(A.H. Sofiyev 2007), studied dynamic buckling of truncated conical shells made of functionally graded materials (FGMs) subjected to a uniform axial compressive load, which is a linear function of time. The material properties of functionally graded shells are assumed to vary continuously through the thickness of the shell. The variation of properties followed an arbitrary distribution in terms of the volume fractions of the constituents. The fundamental relations, the dynamic stability and compatibility equations of functionally graded truncated conical shells are obtained first. Applying Galerkin's method, these equations have been transformed to a pair of time dependent differential equation with variable coefficient and critical parameters obtained using the Runge–Kutta method.

(A.H. Sofiyev et al 2008), studied the vibration and stability of orthotropic conical shells with non-homogeneous material properties under a hydrostatic pressure. At first, the basic relations have been obtained for orthotropic truncated conical shells, Young's moduli and density of which vary continuously in the thickness direction. By applying the Galerkin method to the foregoing equations, the buckling pressure and frequency parameter of truncated conical shells are obtained from these equations. Finally, carrying out some computations, the effects of the variations of conical shell characteristics, the effects of the non-homogeneity and the orthotropy on the critical dimensionless hydrostatic pressure and lowest dimensionless frequency parameter have been studied, when Young's moduli and density vary together and separately.

(B N Singh* and Jibumon B Babu 2008), studied the thermal buckling analysis of laminated conical shell/panel embedded with and without piezoelectric layer subjected to uniform temperature rise based on a higher-order shear deformation theory using the finite element method. The longitudinal and circumferential components of the displacement field are given as a power series of the transverse coordinate and recast in such a manner that the conditions of zero transverse shear stresses are satisfied *a priori*. The effect of stacking sequence, boundary condition, slant ratio and thickness ratio on the thermal buckling temperature has been examined.

In present work, higher order shear deformation theory (HSDT) is analyzed by using analytic solution. The derived differential motion equations are solved using power series technique, then resulting simultaneous equations are solved using MATLAB7. Higher order shear deformation theory results are compared with those from Love's theory derived in present work, and with those published by other researchers who used different theories.

Mathematical Formulation:

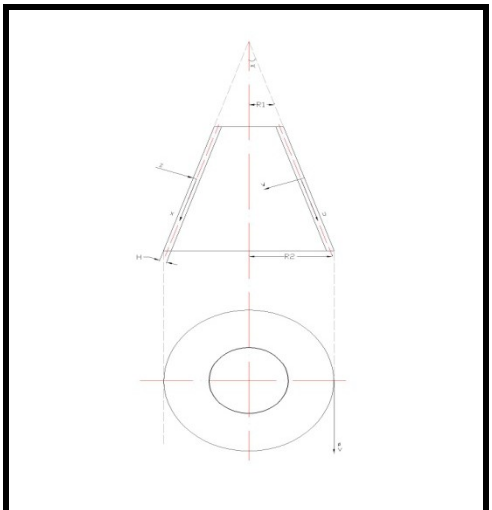

Figure (1): Conical geometry

Displacement components model for (HSDT) theory **(J.N. Reddy 2004)** is as follow (assuming $\varepsilon_4=\varepsilon_5=0$ at $(z=\pm H/2)$), we get:

$$
\begin{aligned}
u(x,\theta,z,t) &= u_0 + z\times\phi_1 - \left(\frac{4}{3H^2}\right)\times z^3\left[\phi_1+\frac{\partial w}{\partial x}\right] \\
v(x,\theta,z,t) &= vo + z\times\phi_2 - \left(\frac{4}{3H^2}\right)\times z^3\left[\phi_2+\left(\frac{1}{R(x)}\right)\left(\frac{\partial w}{\partial\theta}-v_0\cos\alpha\right)\right] \\
w(x,\theta,z,t) &= w_0
\end{aligned} \tag{1}
$$

For conical shell, the strain-displacement relations are:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x}$$
$$\kappa_{xx} = z \times \frac{\partial \phi_1}{\partial x}$$
$$\eta_{xx} = -z^3 \times \left(\frac{4}{3H^2}\right) \times \left(\frac{\partial \phi_1}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right)$$

$$\varepsilon_{\theta\theta} = \left(\frac{1}{R(x)}\right)\left[\frac{\partial v_0}{\partial \theta} + u_0 \sin\alpha + w_0 \cos\alpha\right]$$
$$\kappa_{\theta\theta} = z \times \left(\frac{1}{R(x)}\right)\left[\frac{\partial \phi_2}{\partial \theta} + \sin\alpha \phi_1\right]$$
$$\eta_{\theta\theta} = -z^3 \times \left(\frac{1}{R(x)}\right) \times \left(\frac{4}{3H^2}\right)\left[\begin{array}{l}\frac{\partial \phi_2}{\partial \theta} + \\ \left(\frac{1}{R(x)}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v_0}{\partial \theta}\cos\alpha\right) \\ + \sin\alpha\left(\phi_1 + \frac{\partial w}{\partial x}\right)\end{array}\right]$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} = 0$$

$$\varepsilon_{x\theta} = \varepsilon_6 = \left(\frac{1}{R(x)}\right)\left[\frac{\partial u_0}{\partial \theta} - v_0 \sin\alpha\right] + \frac{\partial v_0}{\partial x}$$
$$\kappa_{x\theta} = z \times \left(\left(\left(\frac{1}{R(x)}\right)\left(\frac{\partial \phi_1}{\partial \theta} - \sin\alpha \frac{\partial \phi_2}{\partial \theta}\right)\right) + \frac{\partial \phi_2}{\partial x}\right)$$
$$\eta_{x\theta} = -z^3 \times \left(\frac{4}{3H^2}\right)\left(\begin{array}{l}\left(\frac{1}{R(x)}\right)\left(\begin{array}{l}\frac{\partial \phi_1}{\partial \theta} + \frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial \phi_2}{\partial \theta} \\ -\left(\frac{1}{R(x)}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \cos\alpha \frac{\partial v_0}{\partial \theta}\right)\end{array}\right) \\ + \frac{\partial \phi_2}{\partial x} - \left(\frac{1}{R(x)}\right)\left(\frac{\partial^2 w}{\partial \theta \partial x} - \cos\alpha \frac{\partial v_0}{\partial x}\right)\end{array}\right)$$

$$\varepsilon_{xz} = \varepsilon_5 = \phi_1 + \frac{\partial w_0}{\partial x}$$
$$\gamma_{xz} = -z^2 \times \left(\frac{4}{H^2}\right)\left(\phi_1 + \frac{\partial w_0}{\partial x}\right)$$

$$\varepsilon_{\theta z} = \varepsilon_4 = \left(\frac{1}{R(x)}\right)\left[\frac{\partial w_0}{\partial \theta} - v_0 \cos\alpha\right] + \phi_2$$
$$\gamma_{\theta z} = -z^2 \times \left(\frac{4}{H^2}\right)\left(\begin{array}{l}\phi_2 + \left(\frac{1}{R(x)}\right)\left(\frac{\partial w_0}{\partial \theta}\right) \\ - v_0 \cos\alpha\end{array}\right) \quad (2)$$

Where: $R(x) = R_o + x \sin\alpha$,,,,,,, α=semi vertex angle. According to Hamilton's Principles **(A.C. Ugral 1981)**:

$$\int_{t2}^{t1} (\delta U - \delta\Omega)\partial t = 0 \quad (3)$$

where:

$$\delta U = \iint_{A\,z} \left(\begin{array}{l}\sigma_{xx}\partial\varepsilon_{xx} + \sigma_{\theta\theta}\partial\varepsilon_{\theta\theta} \\ + \sigma_{zz}\partial\varepsilon_{zz} + \sigma_{x\theta}\partial\varepsilon_{x\theta} \\ + \sigma_{xz}\partial\varepsilon_5 + \sigma_{\theta z}\partial\varepsilon_4\end{array}\right) \times R(x)d\theta dz dx$$

The work done by the direct force, due to displacement (w) only is **(A.C. Ugral 1981)**:

$$\iint_{A\,z} \sigma_{xx}\delta\varepsilon_{xx} R(x)dxd\theta dz = \iint_{A\,z} \sigma_{xx}\delta\left(\begin{array}{l}\frac{\partial u}{\partial x} + \\ z\frac{\partial \phi_1}{\partial x} - \\ \left(\frac{4}{3H^2}\right)\times \\ z^3\left(\begin{array}{l}\frac{\partial \phi_1}{\partial x} \\ + \frac{\partial^2 w}{\partial x^2}\end{array}\right)\end{array}\right) R(x)dx$$
$$= \int_A \left\{\begin{array}{l}N_{xx}\delta\frac{\partial u}{\partial x} \\ + M_{xx}\delta\frac{\partial \phi_1}{\partial x} \\ -\left(\frac{4}{3H^2}\right) \\ \times S_{xx}\delta\left(\frac{\partial \phi_1}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right)R(x)dxd\theta\end{array}\right\}$$

$$\delta\Omega = \frac{1}{2}\int_A\int_z P_{xx} \times \partial\left(\frac{\partial w}{\partial x}\right)^2 \times R(x)\,dz\,dx\,d\theta \qquad (4)$$

where:

$$\int_z \sigma_i\left(1, z, z^3\right)dz = \left(N_i, M_i, S_i\right) \quad \text{(i=x, θ, xθ, θz)}$$

$$\int_z \sigma_i\left(1, z^2\right)dz = \left(Qi, K_i\right) \quad \text{(i=θz, xz)}$$

Substituting eq. ((2) & (4)) in eq. (3) we get 5-equations as follows:

$$\frac{\partial N_{xx}}{\partial x} + \left(N_{xx} - N_{\theta\theta}\right)\left(\frac{\sin\alpha}{R(x)}\right) + \left(\frac{1}{R(x)}\right)\frac{\partial N_{x\theta}}{\partial \theta} = 0$$

$$\left(\frac{1}{R(x)}\right)\frac{\partial N_{\theta\theta}}{\partial \theta} + 2N_{x\theta}\left(\frac{\sin\alpha}{R(x)}\right) + \frac{\partial N_{x\theta}}{\partial x} + \left(\frac{4\cos\alpha}{3H^2 R(x)}\right)\left[\left(\frac{1}{R(x)}\right)\frac{\partial S_{\theta\theta}}{\partial \theta} + 2S_{x\theta}\left(\frac{\sin\alpha}{R(x)}\right) + \frac{\partial S_{\theta z}}{\partial x}\right] + \left(\frac{\cos\alpha}{R(x)}\right)\left[Q_{\theta z} - \left(\frac{4}{H^2}\right)K_{\theta z}\right] = 0$$

$$\left(\frac{4}{3H^2}\right)\left[\left(\frac{2\sin\alpha}{R(x)}\right)\frac{\partial S_{xx}}{\partial x} + \frac{\partial^2 S_{xx}}{\partial x^2} - \frac{\partial S_{\theta\theta}}{\partial x}\left(\frac{\sin\alpha}{R(x)}\right) + \left(\frac{1}{R^2(x)}\right)\frac{\partial^2 S_{\theta\theta}}{\partial \theta^2} + \left(\frac{2}{R(x)}\right)\frac{\partial^2 S_{x\theta}}{\partial x\partial\theta} + \left(\frac{2\sin\alpha}{R^2(x)}\right)\frac{\partial S_{x\theta}}{\partial\theta}\right] - N_{\theta\theta}\left(\frac{\cos\alpha}{R(x)}\right) + Q_{xz}\left(\frac{\sin\alpha}{R(x)}\right) + \frac{\partial Q_{xz}}{\partial x} + \left(\frac{1}{R(x)}\right)\frac{\partial Q_{\theta z}}{\partial\theta} - \left(\frac{4}{H^2}\right)\left[K_{xz}\left(\frac{\sin\alpha}{R(x)}\right) + \frac{\partial K_{xz}}{\partial x} + \left(\frac{1}{R(x)}\right)\frac{\partial K_{\theta z}}{\partial\theta}\right] + \frac{P_{xx}}{2\pi R(x)}\frac{\partial^2 w}{\partial x^2} = 0$$

$$\frac{\partial M_{xx}}{\partial x} - \left(\frac{4}{3H^2}\right)\left[\left(\frac{\sin\alpha}{R(x)}\right)\left(S_{xx} - S_{\theta\theta}\right) + \frac{\partial S_{xx}}{\partial x} + \left(\frac{1}{R(x)}\right)\frac{\partial S_{x\theta}}{\partial\theta}\right] + \left(\frac{\sin\alpha}{R(x)}\right)\left(M_{xx} - M_{\theta\theta}\right) + \left(\frac{1}{R(x)}\right)\frac{\partial M_{x\theta}}{\partial\theta} - Q_{xz} + \left(\frac{4}{H^2}\right)K_{xz} = 0$$

$$\frac{\partial M_{x\theta}}{\partial x} - \left(\frac{4}{3H^2}\right)\left[\left(\frac{2\sin\alpha}{R(x)}\right)S_{x\theta} + \frac{\partial S_{x\theta}}{\partial x} + \left(\frac{1}{R(x)}\right)\frac{\partial S_{\theta\theta}}{\partial\theta}\right] + \left(\frac{2\sin\alpha}{R(x)}\right)M_{x\theta} + \left(\frac{1}{R(x)}\right)\frac{\partial M_{\theta\theta}}{\partial\theta} - Q_{\theta z} + \left(\frac{4}{H^2}\right)K_{\theta z} = 0 \qquad (5)$$

From the constitutive relations of the kth. Lamina the resultants forces-displacement components relations are

$$\begin{Bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \\ M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \\ S_{xx} \\ S_{\theta\theta} \\ S_{x\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \\ B_{11} & B_{12} & B_{16} & C_{11} & C_{12} & C_{16} & E_{11} & E_{12} & E_{16} \\ B_{21} & B_{22} & B_{26} & C_{21} & C_{22} & C_{26} & E_{21} & E_{22} & E_{26} \\ B_{61} & B_{62} & B_{66} & C_{61} & C_{62} & C_{66} & E_{61} & E_{62} & E_{66} \\ D_{11} & D_{12} & D_{16} & E_{11} & E_{12} & E_{16} & H_{11} & H_{12} & H_{16} \\ D_{21} & D_{22} & D_{26} & E_{21} & E_{22} & E_{26} & H_{21} & H_{22} & H_{26} \\ D_{61} & D_{62} & D_{66} & E_{61} & E_{62} & E_{66} & H_{61} & H_{62} & H_{66} \end{bmatrix} \times \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{x\theta} \\ \kappa_{xx} \\ \kappa_{\theta\theta} \\ \kappa_{x\theta} \\ \eta_{xx} \\ \eta_{\theta\theta} \\ \eta_{x\theta} \end{Bmatrix}$$

Also:

$$\begin{Bmatrix} Q_{\theta z} \\ Q_{xz} \\ K_{\theta z} \\ K_{xz} \end{Bmatrix} = \begin{bmatrix} A_{44}A_{45}C_{44}C_{45} \\ A_{54}A_{55}C_{54}C_{55} \\ C_{45}C_{44}E_{45}E_{44} \\ C_{54}C_{55}E_{54}E_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{\theta z} \\ \varepsilon_{xz} \\ \gamma_{\theta z} \\ \gamma_{xz} \end{Bmatrix} \qquad (6)$$

where:

$$\left(A_{ij}, B_{ij}, C_{ij}, D_{ij}, E_{ij}, H_{ij}\right) = \int Q_{ij}\left(1, z, z^2, z^3, z^4, z^5, z^7\right) dz$$

Also:
Substituting eq. (6) in eq. (5) results in:

$$F_{11}U + F_{12}V + F_{13}W + F_{14}\phi^1 + F_{15}\phi^2 = 0 \ldots (a)$$
$$F_{21}U + F_{22}V + F_{23}W + F_{24}\phi^1 + F_{25}\phi^2 = 0 \ldots (b)$$
$$F_{31}U + F_{32}V + F_{33}W + F_{34}\phi^1 + F_{35}\phi^2 + F_N W = 0 \ldots (c)$$
$$F_{41}U + F_{42}V + F_{43}W + F_{44}\phi^1 + F_{45}\phi^2 = 0 \ldots (d)$$
$$F_{51}U + F_{52}V + F_{53}W + F_{54}\phi^1 + F_{55}\phi^2 = 0 \ldots (e) \qquad (7)$$

where the coefficients ($F_{i,\ j}$) are given in Appendix (A). Multiplying eq. (7-(a, d, e)) by $R^3(x)$, eq. (7-(b, c)) by $R^4(x)$ eq. (7) may be modified as:

$$F^*_{11}U + F^*_{12}V + F^*_{13}W + F^*_{14}\phi^1 + F^*_{15}\phi^2 = 0$$
$$F^*_{21}U + F^*_{22}V + F^*_{23}W + F^*_{24}\phi^1 + F^*_{25}\phi^2 = 0$$
$$F^*_{31}U + F^*_{32}V + F^*_{33}W + F^*_{34}\phi^1 + F^*_{35}\phi^2 + F_N{}^* W = 0$$
$$F^*_{41}U + F^*_{42}V + F^*_{43}W + F^*_{44}\phi^1 + F^*_{45}\phi^2 = 0$$
$$F^*_{51}U + F^*_{52}V + F^*_{53}W + F^*_{54}\phi^1 + F^*_{55}\phi^2 = 0 \qquad (8)$$

where: $F^*_{1j}=R^3(x)F_{1j}$ $F^*_{2j}=R^4(x)F_{2j}$ $F^*_{3j}=R^4(x)F_{3j}$ $F^*_{4j}=R^3(x)F_{4j}$ $F^*_{5j}=R^3(x)F_{5j}$ (j=1,2,3,4,5).

Let us assume the solution for eqs. (8) in the following form **(Liyong Tong and Tsun Juei Wang 1992)**:

$$U = u(x)\cos n\theta,,,,,,,, u(x) = \sum_{m=0}^{\infty} a_m x^m$$
$$V = v(x)\sin n\theta,,,,,,,,, v(x) = \sum_{m=0}^{\infty} b_m x^m$$
$$W = w(x)\cos n\theta,,,,,,,, w(x) = \sum_{m=0}^{\infty} c_m x^m$$
$$\phi^1 = \phi_1(x)\cos n\theta,,,,,,,, \phi_1(x) = \sum_{m=0}^{\infty} d_m x^m$$
$$\phi^2 = \phi_2(x)\sin n\theta,,,,,,,, \phi_2(x) = \sum_{m=0}^{\infty} e_m x^m \qquad (9)$$

Where (n) is an integer representing circumferential wave number of the shell, and (a_m, b_m, c_m, d_m, e_m) are constants to be determined later. On substituting from eqs. (9) into eq. (8), five linear algebraic equations are developed, by matching the terms of the same order in x, and further the following recurrence relations are obtained:

$$\begin{aligned} a(m+2) &= J_{1,1}a(m+1) + J_{1,2}a(m) + J_{1,3}a(m-1) \\ &+ J_{1,4}b(m+1) + J_{1,5}b(m) + J_{1,6}b(m-1) + J_{1,7}c(m+3) \\ &+ J_{1,8}c(m+2) + J_{1,9}c(m+1) + J_{1,10}c(m) + J_{1,11}c(m-1) + \\ &J_{1,12}d(m+1) + J_{1,13}d(m) + J_{1,14}d(m-1) + \\ &J_{1,15}e(m+1) + J_{1,16}e(m) + J_{1,17}e(m-1) \ldots\ldots (a) \end{aligned}$$

$$\begin{aligned} b(m+2) &= J_{2,1}a(m+1) + J_{2,2}a(m) + J_{2,3}a(m-1) \\ &+ J_{2,4}a(m-2) + J_{2,5}b(m+1) + J_{2,6}b(m) + J_{2,7}b(m-1) \\ &+ J_{2,8}b(m-2) + J_{2,9}c(m+2) + J_{2,10}c(m+1) + \\ &J_{2,11}c(m) + J_{2,12}c(m-1) + J_{2,13}c(m-2) + J_{2,14}d(m+1) \\ &J_{2,15}d(m) + J_{2,16}d(m-1) + J_{2,17}d(m-2) + J_{2,18}e(m+ \\ &J_{2,19}e(m+1) + J_{2,20}e(m) + J_{2,21}e(m-1) + \\ &J_{2,22}e(m-2) + J_{2,23}e(m-3) \ldots\ldots (b) \end{aligned}$$

$$c(m+4)=J_{3,1}a(m+3)+J_{3,2}a(m+2)+J_{3,3}a(m+1)+ J_{3,4}a(m)+J_{3,5}a(m-1)+J_{3,6}a(m-2)+J_{3,8}b(m+2)+ J_{3,9}b(m+1)+J_{3,10}b(m)+J_{3,11}b(m-1)+J_{3,12}b(m-2) +J_{3,14}c(m+3)+J_{3,15}c(m+2)+J_{3,16}c(m+1)+J_{3,17}c(m) J_{3,18}c(m-1)+J_{3,19}c(m-2)+J_{3,22}d(m+3)+J_{3,23}d(m+ +J_{3,24}d(m+1)+J_{3,25}d(m)+J_{3,26}d(m-1)+J_{3,27}d(m- J_{3,28}d(m-3)+J_{3,29}e(m+2)+J_{3,30}e(m+1)+J_{3,31}e(m)+ J_{3,32}e(m-1)+J_{3,33}e(m-2)+J_{3,34}e(m-3)......(c)$$

$$d(m+2)=J_{4,1}a(m+2)+J_{4,2}a(m+1)+J_{4,3}a(m)+ J_{4,4}a(m-1)+J_{4,5}a(m-2)+J_{4,6}b(m+1)+J_{4,7}b(m)+ J_{4,8}b(m-1)+J_{4,9}c(m+3)+J_{4,10}c(m+2)+J_{4,11}c(m+1) +J_{4,12}c(m)+J_{4,13}c(m-1)+J_{4,14}c(m-2)+ J_{4,15}d(m+1)+J_{4,16}d(m)+J_{4,17}d(m-1)+ J_{4,18}d(m-2)+J_{4,19}d(m-3)+J_{4,20}e(m+1)+J_{4,21}e(m)+ J_{4,22}e(m-1)......(d)$$

Similar to **(Liyong Tong and Tsun Juei Wang 1992)**, the constants a_m, b_m, d_m, e_m (m>=2) and c_m (m>=4) can be expressed in term of (a_0, a_1, b_0, b_1, c_0, c_1, c_2, c_3, d_0, d_1, e_0, e_1,) with recurrence relations in eqs. (10), and are to be determined by imposing boundary conditions at both ends of conical. Also, same procedure in **(Liyong Tong and Tsun Juei Wang 1992)**, the convergence condition for series in eqs. (9) may be obtained as:

$$R_1 \geq 0 \qquad (11)$$

Two types of boundary conditions are used:

1- Simply supported boundary conditions at x=±L
   SS1: $V=N_{xx}=M_{xx}=W=0$
   SS2: $V=U=M_{xx}=W=0$
2- Clamped boundary conditions at x=±L
   CC1: $Nxx=N_{x\varphi}=W=\partial W/\partial x=0$
   CC2: $V=N_{xx}=W=\partial W/\partial x=0$

Displacement components model for Love 's theory **(Soedel W. 2000)** is as follow:

$$u(x,\theta,z,t)=u_0+z\times\beta_1 \\ v(x,\theta,z,t)=vo+z\times\beta_2 \\ w(x,\theta,z,t)=w_0 \qquad (12)$$

Assuming $\varepsilon_4=\varepsilon_5=0$ at (z=±H/2), we get:

$$\beta_1=-\frac{\partial w_0}{\partial x}$$
$$\beta_2=\left(\frac{v_0\cos\alpha}{R(x)}\right)-\frac{1}{R(x)}\left(\frac{\partial w_0}{\partial\theta}\right)$$

For conical shell, the strain-displacement relations are:

$$\varepsilon_{xx}=\frac{\partial u_0}{\partial x}$$
$$\kappa_{xx}=-z\left(\frac{\partial^2 w}{\partial x^2}\right)$$

$$\varepsilon_{\theta\theta}=\left(\frac{1}{R(x)}\right)\left[\frac{\partial v_0}{\partial\theta}+u_0\sin\alpha+w_0\cos\alpha\right]$$
$$\kappa_{\theta\theta}=-z\times\left(\frac{1}{R(x)}\right)\times\left[\left(\frac{1}{R(x)}\right)\left(\frac{\partial^2 w}{\partial\theta^2}\right)+\sin\alpha\left(\frac{\partial w}{\partial x}\right)\right]$$

$$e(m+2)=J_{5,1}a(m+1)+J_{5,2}a(m)+ J_{5,3}a(m-1)+J_{5,4}b(m+2)+ J_{5,5}b(m+1)+J_{5,6}b(m)+ J_{5,7}b(m-1)+J_{5,8}b(m-2)+ J_{5,9}c(m+2)+J_{5,10}c(m+1)+ J_{5,11}c(m)+J_{5,12}c(m-1)+ J_{5,13}c(m-2)+J_{5,14}d(m+1) +J_{5,15}d(m)+J_{5,16}d(m-1)+ J_{5,17}e(m+1)+J_{5,18}e(m)+ J_{5,19}e(m-1)+J_{5,20}e(m-2) +J_{5,21}e(m-3)........(e) \qquad (10)$$

$$\varepsilon_{zz}=\frac{\partial w}{\partial z}=0$$

$$\varepsilon_{x\theta}=\varepsilon_6=\left(\frac{1}{R(x)}\right)\left[\frac{\partial u_0}{\partial\theta}-v_0\sin\alpha\right]+\frac{\partial v_0}{\partial x}$$
$$\kappa_{x\theta}=-2\times z\ \times\left(\begin{array}{c}\left(\frac{1}{R(x)}\right)\left(\frac{\partial^2 w}{\partial x\partial\theta}\right)\\ -\left(\frac{\sin\alpha}{R(x)}\right)\left(\frac{\partial w}{\partial\theta}\right)\end{array}\right) \quad (13)$$

Where: $R(x)=R_o+x\sin\alpha,,,,,$ α= semi vertex angle. According to Hamilton's Principles:

$$\int_{t2}^{t1}(\delta U-\delta\Omega)\partial t=0 \quad (14)$$

where:

$$\delta U=\int_A\int_z\left(\begin{array}{c}\sigma_{xx}\partial\varepsilon_{xx}\\ +\sigma_{\theta\theta}\partial\varepsilon_{\theta\theta}+\\ \sigma_{x\theta}\partial\varepsilon_{x\theta}\end{array}\right)\times R(x)d\theta dzdx$$

The work done by the direct force, due to displacement (w) only is **(A.C. Ugral 1981)**:

$$\delta\Omega=\frac{1}{2}\int_A\int_z P_{xx}\times\partial(\frac{\partial\ w}{\partial x})^2\times R(x)dzdxd\theta$$
$$\int_A\int_z\sigma_{xx}\delta\varepsilon_{xx}R(x)dxd\theta dz=\int_A\int_z\sigma_{xx}\delta\left(\begin{array}{c}\frac{\partial u}{\partial x}\\ -z\\ \left(\frac{\partial^2 w}{\partial x^2}\right)\end{array}\right)R(x)dxd\theta dz$$
$$=\int_A\left\{N_{xx}\delta\frac{\partial u}{\partial x}-M_{xx}\delta\left(\frac{\partial^2 w}{\partial x^2}\right)R(x)dxd\theta\right\} \quad (15)$$

where: $\int_z\sigma_i\left(1,z,z^3\right)dz=\left(N_i,M_i,S_i\right)$

(i=x, θ, xθ). Substituting eq. ((13) & (15)) in eq. (14) we get 3-equations as follows:

$$\frac{\partial N_{xx}}{\partial x}+\left(N_{xx}-N_{\theta\theta}\right)\left(\frac{\sin\alpha}{R(x)}\right)+\left(\frac{1}{R(x)}\right)\frac{\partial N_{x\theta}}{\partial\theta}=0$$

$$\left(\frac{1}{R(x)}\right)\frac{\partial N_{\theta\theta}}{\partial\theta}+2N_{x\theta}\left(\frac{\sin\alpha}{R(x)}\right)+\frac{\partial N_{x\theta}}{\partial x}+$$
$$\left(\frac{\cos\alpha}{R(x)}\right)\left[2M_{x\theta}\left(\frac{\sin\alpha}{R(x)}\right)+\frac{\partial M_{\theta x}}{\partial x}+\left(\frac{1}{R(x)}\right)\frac{\partial M_{\theta\theta}}{\partial\theta}\right]=0$$

$$\left[\begin{array}{c}\left(\frac{2\sin\alpha}{R(x)}\right)\frac{\partial M_{xx}}{\partial x}+\frac{\partial^2 M_{xx}}{\partial x^2}-\frac{\partial M_{\theta\theta}}{\partial x}\left(\frac{\sin\alpha}{R(x)}\right)+\\ \left(\frac{1}{R^2(x)}\right)\frac{\partial^2 M_{\theta\theta}}{\partial\theta^2}+\left(\frac{2}{R(x)}\right)\frac{\partial^2 M_{x\theta}}{\partial x\partial\theta}+\left(\frac{2\sin\alpha}{R^2(x)}\right)\frac{\partial M_{x\theta}}{\partial\theta}\end{array}\right]$$
$$-N_{\theta\theta}\left(\frac{\cos\alpha}{R(x)}\right)+\frac{P_{xx}}{2\pi R(x)}\frac{\partial^2 w}{\partial x^2}=0 \quad (16)$$

From the constitutive relations of the kth. Lamina the resultants forces-displacement components relations are:

$$\left\{\begin{array}{c}N_{xx}\\ N_{\theta\theta}\\ N_{x\theta}\\ M_{xx}\\ M_{\theta\theta}\\ M_{x\theta}\end{array}\right\}=\left[\begin{array}{cccccc}A_{11}&A_{12}&A_{16}&B_{11}&B_{12}&B_{16}\\ A_{21}&A_{22}&A_{26}&B_{21}&B_{22}&B_{26}\\ A_{61}&A_{62}&A_{66}&B_{61}&B_{62}&B_{66}\\ B_{11}&B_{12}&B_{16}&C_{11}&C_{12}&C_{16}\\ B_{21}&B_{22}&B_{26}&C_{21}&C_{22}&C_{26}\\ B_{61}&B_{62}&B_{66}&C_{61}&C_{62}&C_{66}\end{array}\right]\times\left\{\begin{array}{c}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{x\theta}\\ \kappa_{xx}\\ \kappa_{\theta\theta}\\ \kappa_{x\theta}\end{array}\right\} \quad (17)$$

Also:

where: $\left(A_{ij}, B_{ij}, C_{ij}\right) = \int Q_{ij}\left(1, z, z^2\right)dz$.

Also substituting eq. (17) in eq. (16) results in:

$$F_{11}U + F_{12}V + F_{13}W + F_{14}\phi^1 + F_{15}\phi^2 = 0 \ldots\ldots\ldots(a)$$
$$F_{21}U + F_{22}V + F_{23}W + F_{24}\phi^1 + F_{25}\phi^2 = 0 \ldots\ldots\ldots(b)$$
$$F_{31}U + F_{32}V + F_{33}W + F_{34}\phi^1 + F_{35}\phi^2 + F_N W = 0 \ldots\ldots(c) \qquad (18)$$

Multiplying eq. (18-(a)) by $R^3(x)$, eq. (18-(b, c)) by $R^4(x)$ eqs. (18) may be modified as:

$$F_{11}^*U + F_{12}^*V + F_{13}^*W + F_{14}^*\phi^1 + F_{15}^*\phi^2 = 0$$
$$F_{21}^*U + F_{22}^*V + F_{23}^*W + F_{24}^*\phi^1 + F_{25}^*\phi^2 = 0$$
$$F_{31}^*U + F_{32}^*V + F_{33}^*W + F_{34}^*\phi^1 + F_{35}^*\phi^2 + F_N^*W = 0 \qquad (19)$$

where: $F^*_{1j}=R^3(x)F_{1j}$ $F^*_{2j}=R^4(x)F_{2j}$ $F^*_{3j}=R^4(x)F_{3j}$ (j=1,2,3,4,5), let us assume the solution for eqs. (19) in the following form **(Liyong Tong and Tsun Juei Wang 1992)**:

$$U = u(x)\cos n\theta,,,,,,,,,,,,u(x) = \sum_{m=0}^{\infty} a_m x^m$$
$$V = v(x)\sin n\theta,,,,,,,,,,,,v(x) = \sum_{m=0}^{\infty} b_m x^m$$
$$W = w(x)\cos n\theta,,,,,,,,,,,w(x) = \sum_{m=0}^{\infty} c_m x^m \qquad (20)$$

Using the same solution procedure shown in (HSDT), the following recurrence relations are obtained

$$a(m+2) = J_{1,1}a(m+1) + J_{1,2}a(m) + J_{1,3}a(m-1) + J_{1,4}b(m+1) + J_{1,5}b(m) + J_{1,6}b(m-1) + J_{1,7}c(m+3) + J_{1,8}c(m+2) + J_{1,9}c(m+1) + J_{1,10}c(m) + J_{1,11}c(m-1) \ldots\ldots(a)$$

$$c(m+4) = J_{3,1}a(m+3) + J_{3,2}a(m+2) + J_{3,3}a(m+1) + J_{3,4}a(m) + J_{3,5}a(m-1) + J_{3,6}a(m-2) + J_{3,8}b(m+2) + J_{3,9}b(m+1) + J_{3,10}b(m) + J_{3,11}b(m-1) + J_{3,12}b(m-2) + J_{3,14}c(m+3) + J_{3,15}c(m+2) + J_{3,16}c(m+1) + J_{3,17}c(m) + J_{3,18}c(m-1) + J_{3,19}c(m-2) \ldots\ldots(c)$$

Numerical Results:

Buckling of isotropic conical shells under axial compression with different parameters under different boundary conditions, are analyzed. The obtained results ρ_{cr} and their comparison with those in **(Liyong Tong and Tsun Juei Wang 1992)** are shown with different values of (L/R_1), semi vertex angles (α) and different boundary conditions i.e. SS1 in **Table (1)** and SS2 in **Table(2)**. Present work results for ρ_{cr} are close to those from **(Liyong Tong and Tsun Juei Wang 1992)**, where: $\rho_{cr}=(P_{cr}/P_{cl})$ and $P_{cl} = \left(\frac{2\pi E H^2 \cos\alpha^2}{\sqrt{3(1-\mu^2)}}\right)$ (E= modulus of elasticity, H= total shell thickness and μ= Poisson's ratio). There is however a difference in circumferential wave number. It can be seen that ρ_{cr} tends to (.5) for SS1 and to (1) for SS2, so that buckling critical values for SS1 is lower

$$b(m+2) = J_{2,1}a(m+1) + J_{2,2}a(m) + J_{2,3}a(m-1) + J_{2,4}a(m-2) + J_{2,5}b(m+1) + J_{2,6}b(m) + J_{2,7}b(m-1) + J_{2,8}b(m-2) + J_{2,9}c(m+2) + J_{2,10}c(m+1) + J_{2,11}c(m) + J_{2,12}c(m-1) + J_{2,13}c(m-2) \ldots\ldots(b)$$

than that for SS2 shells. For short conical with (L/R_1) =.2, ρ_{cr} becomes large as (α) increase, and it tends to a constant value independent of (α) for cones with (L/R_1) =.5. While for clamped boundaries (CC1& CC2), ρ_{cr} tends to (1) for cones with (L/R_1) =.5 and ρ_{cr} becomes large as (α) increase as shown in **Tables (3)** and **(4).** Results of both HSDT and Loves theory are almost close to each other.

Laminated cross-plied cones critical buckling load are obtained using two different shell theories, under changing different parameters. Critical load ratios and associated circumferential wave numbers are shown for these cones with ($R_1/H=100$), and with different values of (L/R_1), semi vertex angles α, number of laminate and boundary conditions, i. e. SS1 in **Table(5)**, SS2 in **Table(6)**, CC1 in **Table(7)** and CC2 in **Table(8)**. we can see from **Table(5)** to **Table (8)** (as proven by other researchers) that load ratio increase as total number of layer becomes larger for cones with (L/R_1) and α fixed. For length -to- radius ratio (L/R_1) =0.2 the critical load ratio is larger than its value when $(L/R_1)>0.2$which tends to be constant and other parameters remain unchanged.

Another important parameter effect is that of changing of α on load ratio which is similar to that of number of layers (**ρ_{cr}** changes directly with α). However for short cones this effect is very strong, while for long cones it becomes quite weak.

In all above calculations, (35) terms of eqs. (9 & 20) are used and we compute **ρ_{cr}** with μ is replaced by $\mu_{\varphi x}$ and E by E_x for laminated cones.

Table (1): Critical load ratio ρ_{cr} and (n) for SS1 boundary conditions (μ=.3, $R_1/H=100$).

α	Theory	$(L/R_1)=.2$		$(L/R_1)=.5$	
		Present(n)	**(Liyong Tong and Tsun Juei Wang 1992)**	Present(n)	**(Liyong Tong and Tsun Juei Wang 1992)**
10^0	HSDT	.4673(0)	0.5075(0)	.4718(0)	0.5147(0)
	Love 's	.4809(0)		.4896(0)	
30^0	HSDT	.5101(0)	0.5567(0)	.5001(0)	0.5139(0)
	Love 's	.5414(0)		.5165(0)	
60^0	HSDT	.8205(0)	0.8701(0)	.4102(0)	0.4486(0)
	Love 's	.8662(0)		.4408(0)	

Table (2): Critical load ratio ρ_{cr} and (n) for SS2 boundary conditions (μ=.3, $R_1/H=100$).

α	Theory	$(L/R_1)=.2$		$(L/R_1)=.5$	
		Present(n)	**(Liyong Tong and Tsun Juei Wang 1992)**	Present(n)	**(Liyong Tong and Tsun Juei Wang 1992)**
10^0	HSDT	.968(7)	1.007(7)	.931(8)	1.002(8)
	Love 's	1.009(7)		1.002(8)	
30^0	HSDT	1.012(5)	1.017(5)	.989(7)	1.001(7)
	Love 's	1.015(5)		1.003(7)	
60^0	HSDT	1.092(0)	1.144(0)	1.025(5)	1.044(7)
	Love 's	1.113(0)		1.038(5)	

Table (3): Critical load ratio ρ_{cr} and (n) for CC1 boundary conditions (μ=.3, R_1/H=100).					
α	Theory	(L/R_1)=.2		(L/R_1)=.5	
		Present(n)	Discrepancy (%)	Present(n)	Discrepancy (%)
10^0	HSDT	1.6807(0)	8.692	.9735 (1)	3.785
	Love 's	1.8407(0)		1.0118(1)	
30^0	HSDT	1.7743(0)	8.916	0.9015(7)	9.904
	Love 's	1.9480(0)		1.0006(7)	
60^0	HSDT	3.0251(0)	8.855	.9231(0)	8.694
	Love 's	3.3190(0)		1.0110(0)	

Table (4): Critical load ratio ρ_{cr} and (n) for CC2 boundary conditions (μ=.3, R_1/H=100).					
α	Theory	(L/R_1)=.2		(L/R_1)=.5	
		Present(n)	Discrepancy (%)	Present(n)	Discrepancy (%)
10^0	HSDT	1.6311(0)	4.317	.8725(8)	5.327
	Love 's	1.7047(0)		.9216(8)	
30^0	HSDT	1.7851(0)	7.794	.9086(8)	6.310
	Love 's	1.9360(0)		.9698(8)	
60^0	HSDT	3.1405(0)	6.535	1.0012(0)	1.968
	Love 's	3.3601(0)		1.0213(0)	

Table (5): Critical load ratio ρ_{cr} and (n) for multilayered cross-ply with SS1 boundary conditions ($R_1/H=100$).

α	No.	HSDT	Love 's	Discrepancy	HSDT	Love 's	Discrepancy
		$(L/R_1)=0.2$		(%)	$(L/R_1)=0.5$		(%)
10	2	.0821(8)	.0865(8)	5.086	.0537(7)	.0590(7)	8.983
	4	.1780(7)	.1812(7)	1.766	.0815(6)	.0891(6)	8.529
	6	.1863(7)	.1986(7)	6.193	.0911(6)	.0960(6)	5.104
30	2	.0901(8)	.0923(8)	2.383	.0554(7)	.0592(7)	6.418
	4	.2001(6)	.2131(6)	6.100	.0907(6)	.0933(6)	2.786
	6	.2142(6)	.2212(6)	3.164	.0935(5)	.0962(5)	2.806
60	2	.1275(6)	.1381(6)	7.675	.0567(5)	.0595(5)	4.705
	4	.3087(5)	.3159(5)	2.279	.1008(4)	.1087(4)	7.267
	6	.3363(5)	.3551(5)	5.294	.1053(4)	.1091(4)	3.483

Table (6): Critical load ratio ρ_{cr} and (n) for multilayered cross-ply with SS2 boundary conditions ($R_1/H=100$).

α	No.	HSDT	Love 's	Discrepancy	HSDT	Love 's	Discrepancy
		$(L/R_1)=0.2$		(%)	$(L/R_1)=0.5$		(%)
10	2	.1522(9)	.1636(9)	6.968	.0764(9)	.0790(9)	3.291
	4	.2011(8)	.2206(8)	8.839	.1075(6)	.1105(6)	2.714
	6	.2095(8)	.2267(8)	7.587	.1082(6)	.1119(6)	3.306
30	2	.1706(9)	.1818(9)	6.160	.0803(9)	.0828(9)	3.019
	4	.2261(8)	.2451(8)	7.751	.1071(6)	.1108(6)	3.339
	6	.2278(8)	.2497(8)	8.770	.1086(6)	.1121(6)	3.122
60	2	.2661(7)	.2901(7)	8.273	.0915(6)	.0959(6)	4.588
	4	.3682(6)	.3781(6)	2.618	.1121(5)	.1230(5)	8.861
	6	.3706(6)	.3864(6)	4.089	.1167(5)	.1251(5)	6.714

Table (7): Critical load ratio ρ_{cr} and (n) for multilayered cross-ply with CC1 boundary conditions ($R_1/H=100$).							
α	No.	HSDT	Love 's	Discrepancy (%)	HSDT	Love 's	Discrepancy (%)
		$(L/R_1)=0.2$			$(L/R_1)=0.5$		
10	2	.2418(9)	.2616(9)	7.568	.0862(8)	.0893 (8)	3.471
	4	.6001(8)	.6341(8)	5.361	.1541(6)	.1673(6)	7.890
	6	.7027(8)	.7116(8)	1.250	.1633(6)	.1791(6)	8.821
30	2	.2766(9)	.2935(9)	5.758	.0901(7)	.0912(7)	1.206
	4	.7057(7)	.7136(7)	1.107	.1672(6)	.1761(6)	5.053
	6	.8023(7)	.8146(7)	1.509	.1705(6)	.1887(6)	9.644
60	2	.4721(7)	.4913(7)	3.907	.1081(6)	.1151(6)	6.081
	4	1.1061(5)	1.2103(5)	8.609	.2282(5)	.2470(5)	7.611
	6	1.2858(5)	1.3377(5)	3.879	.2508(5)	.2641(5)	5.035

Table (8): Critical load ratio ρ_{cr} and (n) for multilayered cross-ply with CC2 boundary conditions ($R_1/H=100$).							
α	No.	HSDT	Love 's	Discrepancy (%)	HSDT	Love 's	Discrepancy (%)
		$(L/R_1)=0.2$			$(L/R_1)=0.5$		
10	2	.2501 (9)	.2618(9)	4.469	.0851(8)	.0887 (8)	4.058
	4	.6167(8)	.6282(8)	1.830	.1507(6)	.1641(6)	8.165
	6	.7061(8)	.7141(8)	1.120	.1663(6)	.1791(6)	7.146
30	2	.2681(9)	.2874(9)	6.715	.0924(7)	.0943(7)	2.014
	4	.7015(8)	.7153(8)	1.929	.1602(6)	.1726(6)	7.184
	6	.7601(8)	.7905(8)	3.845	.1782(6)	.1889(6)	5.664
60	2	.4601(7)	.4812(7)	4.384	.1095(6)	.1153(6)	5.030
	4	1.166(6)	1.2116(6)	3.763	.2221(5)	.2437(5)	8.863
	6	1.3191(6)	1.3409(6)	1.625	.2413(5)	.2645(5)	8.771

Conclusions:

Analytical solution for buckling of Loves and third-order shell theories are applied to cross-ply cone shells. Third-order shell theory yields results close to those of Love shell theory but it almost under predicts buckling loads, the maximum discrepancy is (9.904%). For simply supported shells buckling parameter (**ρ_{cr}**) changes directly with the number of layers for same (α, (L/R_1)), also semi vertex angle has the same effect on the buckling parameter for same (No. , (L/R_1)), while this parameter changes indirectly with (L/R_1) for the same (α, No.). Buckling parameter (**ρ_{cr}**) for clamped boundary conditions is larger than that for simply supported shells. The effect of semi vertex angle on the buckling parameter of long conical shells is less than its effect on short shells.

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Appendix (A):

$$F_{11} = A_{11}\left(\frac{\partial^2}{\partial x^2} + \left(\frac{\sin\alpha}{R(x)}\right)\frac{\partial}{\partial x}\right) - A_{22}\left(\frac{\sin^2\alpha}{R^2(x)}\right) + \left(\frac{A_{66}}{R^2(x)}\right)\frac{\partial^2}{\partial\theta^2}$$

$$F_{12} = \left(\frac{A_{12}+A_{66}}{R(x)}\right)\frac{\partial^2}{\partial x\partial\theta} - \left(\frac{A_{22}+A_{66}}{R^2(x)}\right)\sin\alpha\frac{\partial}{\partial\theta} - \left(\frac{4}{3H^2}\right)\left(\frac{D_{22}+2D_{66}+D_{12}}{R^3(x)}\right)\sin\alpha\cos\alpha\frac{\partial}{\partial\theta} + \left(\frac{4}{3H^2}\right)\left(\frac{D_{66}+D_{12}}{R^2(x)}\right)\cos\alpha\frac{\partial^2}{\partial x\partial\theta}$$

$$F_{13} = \left(\frac{-4}{3H^2}\right)D_{11}\frac{\partial^3}{\partial x^3} + \left(\frac{A_{12}\cos\alpha}{R(x)} + \frac{4D_{22}\sin^2\alpha}{3H^2R^2(x)}\right)\frac{\partial}{\partial x} - \left(\frac{4}{3H^2}\right)\left(\frac{D_{12}+2D_{66}}{R^2(x)}\right)\frac{\partial^3}{\partial x\partial\theta^2} + \left(\frac{4}{3H^2}\right)\left(\frac{D_{12}+2D_{66}+D_{22}}{R^3(x)}\right)\sin\alpha\frac{\partial^2}{\partial\theta^2} - \left(\frac{4}{3H^2}\right)\left(\frac{D_{11}}{R(x)}\right)\sin\alpha\frac{\partial^2}{\partial x^2} - \left(\frac{A_{22}}{R^2(x)}\right)\sin\alpha\cos\alpha$$

$$F_{14} = \left(B_{11} - \left(\frac{4}{3H^2}\right)D_{11}\right)\frac{\partial^2}{\partial x^2} + \left(B_{11} - \left(\frac{4}{3H^2}\right)D_{11}\right)\left(\frac{\sin\alpha}{R(x)}\right)\frac{\partial}{\partial x} + \left(-B_{22} + \left(\frac{4}{3H^2}\right)D_{22}\right)\left(\frac{\sin^2\alpha}{R^2(x)}\right) + \left(B_{66} - \left(\frac{4}{3H^2}\right)D_{66}\right)\left(\frac{1}{R^2(x)}\right)\frac{\partial^2}{\partial\theta^2}$$

$$F_{15}=\left(\frac{1}{R(x)}\right)\left(B_{12}+B_{66}-\left(\frac{4}{3H^2}\right)(D_{12}+D_{66})\right)\frac{\partial^2}{\partial x\partial\theta}$$
$$-\left(\frac{\sin\alpha}{R^2(x)}\right)\left(B_{22}+B_{66}-\left(\frac{4}{3H^2}\right)(D_{22}+D_{66})\right)\frac{\partial}{\partial\theta}$$

$$F_{21}=\left[\left(\frac{1}{R(x)}\right)(A_{21}+A_{66})+\left(\frac{4\cos\alpha}{3H^2R^2(x)}\right)\begin{pmatrix}D_{21}\\+D_{66}\end{pmatrix}\right]$$
$$+\left[\left(\frac{\sin\alpha}{R^2(x)}\right)(A_{22}+A_{66})+\left(\frac{4\cos\alpha\sin\alpha}{3H^2R^3(x)}\right)\begin{pmatrix}D_{22}\\+D_{66}\end{pmatrix}\right]$$

$$F_{22}=\left[\begin{matrix}\left(\frac{A_{22}}{R^2(x)}\right)+\left(\frac{4\cos\alpha}{3H^2R^3(x)}\right)\\ \left(2D_{22}+\left(\frac{4\cos\alpha}{3H^2R(x)}\right)H_{22}\right)\end{matrix}\right]\frac{\partial^2}{\partial\theta^2}$$
$$+\left[\left(\frac{A_{66}\sin\alpha}{R^2(x)}\right)+\left(\frac{16\cos^2\alpha\sin\alpha}{9H^4R^3(x)}\right)H_{66}\right]\frac{\partial}{\partial x}$$
$$\left[\begin{matrix}\left(\frac{-A_{66}\sin^2\alpha}{R^2(x)}\right)+\left(\frac{4\cos\alpha\sin^2\alpha}{3H^2R^3(x)}\right)D_{66}+\\ \left(\frac{\cos^2\alpha}{R^2(x)}\right)\left(-A_{44}+\left(\frac{8}{H^2}\right)C_{44}-\left(\frac{16}{H^4}\right)E_{44}\right)\end{matrix}\right]+$$
$$\left[A_{66}+\left(\frac{8\cos\alpha}{3H^2R(x)}\right)D_{66}+\left(\frac{16\cos^2\alpha}{9H^4R^2(x)}\right)H_{66}\right]\frac{\partial^2}{\partial x^2}$$

$$F_{23}=\left[\left(\frac{\cos\alpha}{R^2(x)}\right)\begin{pmatrix}A_{22}+A_{44}-\left(\frac{8}{H^2}\right)C_{44}\\+\left(\frac{16}{H^4}\right)E_{44}\end{pmatrix}\right]\frac{\partial}{\partial\theta}$$
$$+\left[\left(\frac{-4\sin\alpha}{3H^2R^2(x)}\right)\left(D_{22}+\left(\frac{4\cos\alpha}{3H^2R(x)}\right)H_{22}\right)\right]\frac{\partial^2}{\partial x\partial\theta}+$$
$$\left[\left(\frac{-4}{3H^2R(x)}\right)\begin{pmatrix}D_{21}+2D_{66}+\\ \left(\frac{4\cos\alpha}{3H^2R(x)}\right)(H_{22}+2H_{66})\end{pmatrix}\right]\frac{\partial^3}{\partial x^2\partial\theta}$$
$$+\left[\left(\frac{-4}{3H^2R^3(x)}\right)\left(D_{22}+\left(\frac{4\cos\alpha}{3H^2R(x)}\right)H_{22}\right)\right]\frac{\partial^3}{\partial\theta^3}$$

$$F_{24}=\left[\begin{matrix}\left(\frac{1}{R(x)}\right)(B_{21}+B_{66})+\left(\frac{4}{3H^2R(x)}\right)(-D_{21}-D_{66})\\ +\left(\frac{4\cos\alpha}{3H^2R^2(x)}\right)\left(E_{21}+E_{66}+\left(\frac{4}{3H^2}\right)(-H_{21}-H_{66})\right)\end{matrix}\right]\frac{\partial^2}{\partial x\partial\ldots}$$
$$\left[\begin{matrix}\left(\frac{\sin\alpha}{R^2(x)}\right)(B_{22}+B_{66})+\left(\frac{4\sin\alpha}{3H^2R^2(x)}\right)(-D_{22}-D_{66})\\ +\left(\frac{4\cos\alpha\sin\alpha}{3H^2R^3(x)}\right)\left(E_{22}+E_{66}+\left(\frac{4}{3H^2}\right)(-H_{22}-H_{66})\right)\end{matrix}\right]\frac{\partial}{\partial\theta}$$

$$F_{25}=\left[\left(\frac{B_{22}}{R^2(x)}\right)+\left(\frac{4}{3H^2R^2(x)}\right)\begin{pmatrix}-D_{22}+\left(\frac{\cos\alpha E_{22}}{R(x)}\right)\\-\left(\frac{4H_{22}\cos\alpha}{3H^2}\right)\end{pmatrix}\right]\frac{\partial^2}{\partial\theta^2}+$$
$$\left[\left(\frac{B_{66}\sin\alpha}{R(x)}\right)+\left(\frac{4\sin\alpha}{3H^2R(x)}\right)\begin{pmatrix}-D_{66}+\left(\frac{\cos\alpha E_{66}}{R(x)}\right)\\-\left(\frac{4H_{66}\cos\alpha}{3H^2R(x)}\right)\end{pmatrix}\right]\frac{\partial}{\partial x}+$$
$$\left[\begin{matrix}\left(\frac{-B_{66}\sin^2\alpha}{R^2(x)}\right)+\left(\frac{4\sin^2\alpha}{3H^2R^2(x)}\right)\left(\frac{4H_{66}\cos\alpha}{3H^2R(x)}+D_{66}\right)\\ +\left(\frac{\cos\alpha}{R(x)}\right)\left(A_{44}-\left(\frac{8}{H^2}\right)C_{44}+\left(\frac{16}{H^4}\right)E_{44}\right)\end{matrix}\right]+$$
$$\left[B_{66}-\left(\frac{4D_{66}}{3H^2}\right)+\left(\frac{4\cos\alpha}{3H^2R(x)}\right)\left(E_{66}-\left(\frac{4}{3H^2}\right)H_{66}\right)\right]\frac{\partial^2}{\partial x^2}$$

$$F_{31}=\left(\frac{8D_{11}\sin\alpha}{3H^2R(x)}\right)\frac{\partial^2}{\partial x^2}+\left[\left(-\frac{A_{21}\cos\alpha}{R(x)}\right)+\left(\frac{4D_{22}\sin^2\alpha}{3H^2R^2(x)}\right)\right]\ldots$$
$$+\left[\left(\frac{-A_{22}\cos\alpha\sin\alpha}{R^2(x)}\right)+\left(\frac{4D_{22}\sin^3\alpha}{3H^2R^3(x)}\right)\right]+$$
$$\left(\frac{4D_{11}}{3H^2}\right)\frac{\partial^3}{\partial x^3}+\left[\left(\frac{4D_{21}}{3H^2R^2(x)}\right)+\left(\frac{8D_{66}}{3H^2R^2(x)}\right)\right]\frac{\partial^3}{\partial x\partial\theta^2}$$
$$+\left(\frac{4D_{22}\sin\alpha}{3H^2R^3(x)}\right)\frac{\partial^2}{\partial\theta^2}$$

$$F_{32}=-\left[\left(\frac{4\sin\alpha}{3H^2R^2(x)}\right)\left(\frac{4\cos\alpha}{3H^2R(x)}\right)(2H_{12}+H_{22}+4H_{66})+D_{22}\right]\frac{\partial^2}{\partial x\partial\theta}-$$

$$\left[\begin{array}{l}\left(\frac{4\sin\alpha}{3H^2R^3(x)}\right)\left(\frac{4\sin\alpha\cos\alpha}{3H^2R(x)}\right)(2H_{12}+2H_{22}+4H_{66})+D_{22}\\ -\left(\frac{\cos\alpha}{R^2(x)}\right)(A_{22}+A_{44})+\left(\frac{8\cos\alpha}{H^2R^2(x)}\right)\left(C_{44}-\left(\frac{2E_{44}}{H^2}\right)\right)\end{array}\right]\frac{\partial}{\partial\theta}+$$

$$\left[\left(\frac{B_{66}\sin\alpha}{R(x)}\right)+\left(\frac{4\sin\alpha}{3H^2R(x)}\right)\left(\begin{array}{l}-D_{66}+\left(\frac{\cos\alpha E_{66}}{R(x)}\right)\\ -\left(\frac{4H_{66}\cos\alpha}{3H^2R(x)}\right)\end{array}\right)\right]\frac{\partial}{\partial x}+$$

$$\left[\begin{array}{l}\left(\frac{-B_{66}\sin^2\alpha}{R^2(x)}\right)+\left(\frac{4\sin^2\alpha}{3H^2R^2(x)}\right)\left(\frac{4H_{66}\cos\alpha}{3H^2R(x)}+D_{66}\right)+\\ \left(\frac{\cos\alpha}{R(x)}\right)\left(A_{44}-\left(\frac{8}{H^2}\right)C_{44}+\left(\frac{16}{H^4}\right)E_{44}\right)\end{array}\right]+$$

$$\left[\left(\frac{4}{3H^2}\right)(D_{12}+2D_{66})+\left(\frac{16\cos\alpha}{9H^4R^2(x)}\right)(H_{12}+2H_{66})\right]\frac{\partial^3}{\partial x^2\partial\theta}+$$

$$\left[\left(\frac{4}{3H^2R^3(x)}\right)\left(D_{22}+\left(\frac{4\cos\alpha}{3H^2R(x)}\right)H_{22}\right)\right]\frac{\partial^3}{\partial\theta^3}$$

$$F_{33}=\left(\frac{-32H_{11}\sin\alpha}{9H^4R(x)}\right)\frac{\partial^3}{\partial x^3}+\left[\begin{array}{l}\left(\frac{A_{55}\sin\alpha}{R(x)}\right)+\left(\frac{8\sin\alpha}{H^2R(x)}\right)\left(\begin{array}{l}-C_{55}\\ +\left(\frac{2}{H^2}\right)E_{55}\end{array}\right)-\\ \left(\frac{16H_{22}\sin^3\alpha}{9H^4R^3(x)}\right)\end{array}\right]\frac{\partial}{\partial x}$$

$$+\left[\left(\frac{-A_{22}\cos^2\alpha}{R^2(x)}\right)+\left(\frac{4D_{22}\sin^2\alpha\cos\alpha}{3H^2R^3(x)}\right)\right]+$$

$$\left[\left(\frac{8D_{12}\cos\alpha}{3H^2R(x)}\right)+\left(\frac{16H_{22}\sin^2\alpha}{9H^4R^2(x)}\right)+A_{55}+\left(\frac{8}{H^2}\right)\left(-C_{55}+\left(\frac{2}{H^2}\right)E_{55}\right)\right]\frac{\partial^2}{\partial x^2}$$

$$\left[\begin{array}{l}\left(\frac{8D_{22}\cos\alpha}{3H^2R^3(x)}\right)-\left(\frac{32\sin^2\alpha}{9H^4R^4(x)}\right)(H_{12}+H_{22}+2H_{66})\\ +\left(\frac{1}{R^2(x)}\right)\left(A_{44}+\left(\frac{8}{H^2}\right)\left(-C_{44}+\left(\frac{2}{H^2}\right)E_{44}\right)\right)\end{array}\right]\frac{\partial^2}{\partial\theta^2}-$$

$$-\left[\left(\frac{32H_{21}}{9H^4R^2(x)}\right)+\left(\frac{64H_{66}}{9H^4R^2(x)}\right)\right]\frac{\partial^4}{\partial x^2\partial\theta^2}-\left(\frac{16H_{11}}{9H^4}\right)\frac{\partial^4}{\partial x^4}$$

$$-\left(\frac{16H_{22}}{9H^4R^4(x)}\right)\frac{\partial^4}{\partial\theta^4}+\left[\left(\frac{32H_{21}\sin\alpha}{9H^4R^3(x)}\right)+\left(\frac{64H_{66}\sin\alpha}{9H^4R^3(x)}\right)\right]\frac{\partial^3}{\partial x\partial\theta^2}$$

$$F_{34}=\left[\left(\frac{-32H_{11}}{9H^4R(x)}\right)+\left(\frac{8E_{11}}{3H^2R(x)}\right)\right]\frac{\partial^2}{\partial x^2}+\left[\left(\frac{4E_{11}}{3H^2}\right)-\left(\frac{16H_{11}}{9H^4}\right)\right]\frac{\partial^3}{\partial x^3}$$

$$\left[\begin{array}{l}A_{55}+\left(\frac{8}{H^2}\right)\left(-C_{55}+\left(\frac{2}{H^2}\right)E_{55}\right)+\left(\frac{4\sin^2\alpha}{3H^2R^2(x)}\right)\left(-E_{22}+\left(\frac{4}{3H^2}\right)H_{22}\right)\\ +\left(\frac{\cos\alpha}{R(x)}\right)\left(-B_{21}+\left(\frac{4}{3H^2}\right)D_{21}\right)\end{array}\right]\frac{\partial}{\partial x}$$

$$+\left[\begin{array}{l}\left(E_{22}+\frac{-4H_{22}\cos^2\alpha}{3H^2}\right)\left(\frac{4\sin^3\alpha}{3H^2R^3(x)}\right)+\left(\frac{\sin\alpha\cos\alpha}{R^2(x)}\right)\left(-B_{22}+\left(\frac{4D_{22}}{3H^2}\right)\right)\\ +\left(\frac{\sin\alpha}{R(x)}\right)\left(A_{55}+\left(\frac{8}{H^2}\right)\left(-C_{55}+\left(\frac{2}{H^2}\right)E_{55}\right)\right)\end{array}\right]$$

$$+\left[\left(\frac{4}{3H^2R^2(x)}\right)\left(E_{21}+2E_{66}-\left(\frac{4}{3H^2}\right)(H_{21}+2H_{66})\right)\right]\frac{\partial^3}{\partial x\partial\theta^2}$$

$$+\left[\left(\frac{4E_{22}\sin\alpha}{3H^2R^3(x)}\right)-\left(\frac{16H_{22}\sin\alpha}{9H^4R^3(x)}\right)\right]\frac{\partial^2}{\partial\theta^2}$$

$$F_{35}=\left[\left(\frac{16H_{22}\sin\alpha}{9H^4R^2(x)}\right)-\left(\frac{4E_{22}\sin\alpha}{3H^2R^2(x)}\right)\right]\frac{\partial^2}{\partial x\partial\theta}$$

$$+\left[\left(\frac{4}{3H^2R(x)}\right)\left(E_{21}+2E_{66}-\left(\frac{4}{3H^2}\right)(H_{21}+2H_{66})\right)\right]\frac{\partial^3}{\partial x^2\partial\theta}+$$

$$\left[\begin{array}{l}\left(\frac{1}{R(x)}\right)\left(A_{44}+\left(\frac{8}{H^2}\right)\left(-C_{44}+\left(\frac{2}{H^2}\right)E_{44}\right)\right)\\ +\left(\frac{4\sin^2\alpha}{3H^2R^3(x)}\right)\left(E_{22}-\left(\frac{4}{3H^2}\right)H_{22}\right)\\ +\left(\frac{\cos\alpha}{R^2(x)}\right)\left(-B_{22}+\left(\frac{4}{3H^2}\right)D_{22}\right)\end{array}\right]\frac{\partial}{\partial\theta}$$

$$+\left[\left(\frac{4E_{22}}{3H^2R^3(x)}\right)-\left(\frac{16H_{22}}{9H^4R^3(x)}\right)\right]\frac{\partial^3}{\partial\theta^3}$$

$$F_N=\frac{P_{xx}}{2\pi R(x)}\frac{\partial^2}{\partial x^2}$$

$$F_{41}=\left[\left(\frac{B_{11}\sin\alpha}{R(x)}\right)-\left(\frac{4D_{11}\sin\alpha}{3H^2R(x)}\right)\right]\frac{\partial}{\partial x}+$$

$$\left[\left(\frac{\sin^2\alpha}{R^2(x)}\right)\left(-B_{22}+\left(\frac{4D_{22}}{3H^2}\right)\right)\right]+\left(B_{11}-\left(\frac{4D_{11}}{3H^2}\right)\right)\frac{\partial^2}{\partial x^2}$$

$$+\left[\left(\frac{-4D_{66}}{3H^2R^2(x)}\right)+\left(\frac{B_{66}}{R^2(x)}\right)\right]\frac{\partial^2}{\partial\theta^2}$$

$$F_{42}=\left[\begin{array}{l}\left(\frac{16H_{12}\cos\alpha\sin\alpha}{9H^4R^3(x)}\right)-\left(\frac{4E_{12}\cos\alpha\sin\alpha}{3H^2R^3(x)}\right)\\ +\left(\frac{4\sin\alpha}{3H^2R^2(x)}\right)\left(D_{22}+D_{66}-\left(\frac{4\cos\alpha}{3H^2R(x)}\right)\left(\begin{array}{l}H_{22}+2H_{66}\\ -E_{22}-E_{66}\end{array}\right)\right)\\ +\left(\frac{\sin\alpha}{R^2(x)}\right)\left(-B_{22}-B_{66}\right)\end{array}\right]\frac{\partial}{\partial\theta}+$$

$$\left[\begin{array}{l}-\left(\frac{16H_{12}\cos\alpha}{9H^4R^2(x)}\right)+\left(\frac{4E_{12}\cos\alpha}{3H^2R^2(x)}\right)+\\ \left(\frac{4}{3H^2R(x)}\right)\left(-D_{12}-D_{66}-\left(\frac{4\cos\alpha}{3H^2R(x)}\right)\left(-H_{66}+E_{66}\right)\right)\\ +\left(\frac{1}{R(x)}\right)\left(B_{12}+B_{66}\right)\end{array}\right]\frac{\partial^2}{\partial x\partial\theta}$$

$$F_{43}=\left[\left(\frac{16H_{11}\sin\alpha}{9H^4R(x)}\right)-\left(\frac{4E_{11}\sin\alpha}{3H^2R(x)}\right)\right]\frac{\partial^2}{\partial x^2}+$$

$$\left[\left(\frac{\sin\alpha\cos\alpha}{R^2(x)}\right)\left(\left(\frac{4}{3H^2}\right)\left(D_{22}-B_{22}\right)\right)\right]+$$

$$\left[\begin{array}{l}\left(-A_{55}+\left(\frac{8}{H^2}\right)\left(C_{55}-\left(\frac{2}{H^2}\right)E_{55}\right)\right)+\\ \left(\frac{4\sin^2\alpha}{3H^2R^2(x)}\right)\left(E_{22}-\left(\frac{4}{3H^2}\right)H_{22}\right)+\\ \left(\frac{\cos\alpha}{R(x)}\right)\left(B_{12}-\left(\frac{4}{3H^2}\right)D_{12}\right)\end{array}\right]\frac{\partial}{\partial x}+$$

$$\left[\left(\frac{4\sin\alpha}{3H^2R^3(x)}\right)\left(\begin{array}{l}E_{22}+E_{12}+2E_{66}+\\ \left(\frac{4}{3H^2}\right)\left(-H_{12}-H_{22}-2H_{66}\right)\end{array}\right)\right]\frac{\partial^2}{\partial\theta^2}$$

$$+\left[\left(-\frac{4E_{11}}{3H^2}\right)+\left(\frac{16H_{11}}{9H^4}\right)\right]\frac{\partial^3}{\partial x^3}+$$

$$\left[\left(\frac{4\sin\alpha}{3H^2R^3(x)}\right)\left(\begin{array}{l}-E_{12}-2E_{66}\\ +\left(\frac{4}{3H^2}\right)\left(H_{12}+2H_{66}\right)\end{array}\right)\right]\frac{\partial^3}{\partial x\partial\theta^2}$$

$$F_{44}=\left[\begin{array}{l}\left(\frac{16H_{11}\sin\alpha}{9H^4R(x)}\right)-\left(\frac{8E_{11}\sin\alpha}{3H^2R(x)}\right)\\ +\left(\frac{\sin\alpha}{R(x)}\right)C_{11}\end{array}\right]\frac{\partial}{\partial x}+$$

$$\left[\begin{array}{l}\left(-A_{55}+\left(\frac{8}{H^2}\right)\left(C_{55}-\left(\frac{2}{H^2}\right)E_{55}\right)\right)+\\ \left(\frac{4\sin^2\alpha}{3H^2R^2(x)}\right)\left(2E_{22}-\left(\frac{4}{3H^2}\right)H_{22}\right)-\left(\frac{\sin^2\alpha}{R^2(x)}\right)C_{22}\end{array}\right]+$$

$$\left[\left(\frac{4}{3H^2}\right)\left(-2E_{11}+\left(\frac{4}{3H^2}\right)H_{11}\right)+C_{11}\right]\frac{\partial^2}{\partial x^2}+$$

$$\left[\left(\frac{4}{3H^2R^2(x)}\right)\left(-2E_{66}+\left(\frac{4}{3H^2}\right)H_{66}\right)+\frac{C_{66}}{R^2(x)}\right]\frac{\partial^2}{\partial\theta^2}$$

$$F_{45}=\left[\begin{array}{l}\left(\frac{4\sin\alpha}{3H^2R^2(x)}\right)\left(\begin{array}{l}2E_{22}+2E_{66}-\\ \left(\frac{4}{3H^2}\right)\left(H_{22}+H_{66}\right)\end{array}\right)\\ -\left(\frac{\sin\alpha}{R^2(x)}\right)\left(C_{22}+C_{66}\right)\end{array}\right]\frac{\partial}{\partial\theta}+$$

$$\left[\begin{array}{l}\left(\frac{4}{3H^2R(x)}\right)\left(\begin{array}{l}-2E_{12}-2E_{66}+\\ \left(\frac{4}{3H^2}\right)\left(H_{12}+H_{66}\right)\end{array}\right)\\ +\left(\frac{1}{R(x)}\right)\left(C_{66}+C_{12}\right)\end{array}\right]\frac{\partial^2}{\partial x\partial\theta}$$

$$F_{51}=\left[\left(\frac{4\sin\alpha}{3H^2R^2(x)}\right)\left(D_{22}-D_{66}\right)-\left(\frac{\sin\alpha}{R^2(x)}\right)\left(B_{22}+B_{66}\right)\right]\frac{\partial}{\partial\theta}$$

$$+\left[\left(\frac{4}{3H^2R(x)}\right)\left(-D_{12}-D_{66}\right)+\left(\frac{1}{R(x)}\right)\left(B_{66}+B_{12}\right)\right]\frac{\partial^2}{\partial x\partial\theta}$$

$$F_{52}=\left[\left(\frac{4\sin\alpha}{3H^2R(x)}\right)\left(\begin{array}{l}-D_{66}-E_{66}\dfrac{\cos\alpha}{R(x)}\\+\left(\dfrac{4\cos\alpha}{3H^2R(x)}\right)(H_{66})\end{array}\right)+\left(\frac{\sin\alpha}{R(x)}\right)(B_{66})\right]\frac{\partial}{\partial x}+$$

$$\left[\left(\frac{4\sin^2\alpha}{3H^2R^2(x)}\right)(D_{66})-\left(\frac{\sin^2\alpha}{R^2(x)}\right)(B_{66})+\left(\frac{\cos\alpha}{R(x)}\right)\left(A_{44}-\left(\frac{8}{H^2R(x)}\right)\left(C_{44}+\left(\frac{4}{H^2}\right)E_{44}\right)\right)\right]+$$

$$\left[\left(\frac{4}{3H^2}\right)\left(\begin{array}{l}-D_{66}+E_{66}\dfrac{\cos\alpha}{R(x)}+\\\left(\dfrac{4\cos\alpha}{3H^2R(x)}\right)(H_{66})\end{array}\right)+(B_{66})\right]\frac{\partial^2}{\partial x^2}+$$

$$\left[\left(\frac{4}{3H^2R^2(x)}\right)\left(\begin{array}{l}-D_{22}+E_{22}\dfrac{\cos\alpha}{R(x)}\\-\left(\dfrac{4\cos\alpha}{3H^2R(x)}\right)(H_{22})\end{array}\right)+\left(\frac{B_{22}}{R^2(x)}\right)\right]\frac{\partial}{\partial\theta}$$

$$F_{53}=\left[\left(\frac{4\sin\alpha}{3H^2R^2(x)}\right)\left(-E_{22}+\left(\frac{4}{3H^2}\right)(H_{22})\right)\right]\frac{\partial^2}{\partial x\partial\theta}$$

$$+\left[\left(\frac{4}{3H^2R^3(x)}\right)\left(-E_{22}+\left(\frac{4}{3H^2}\right)(H_{22})\right)\right]\frac{\partial^3}{\partial\theta^3}$$

$$\left[\left(\frac{4\cos\alpha}{3H^2R^2(x)}\right)(-D_{22})+\left(\frac{\cos\alpha}{R^2(x)}\right)(B_{22})-\left(\frac{1}{R(x)}\right)\left(A_{44}-\left(\frac{8}{H^2R(x)}\right)\left(C_{44}+\left(\frac{4}{H^2}\right)E_{44}\right)\right)\right]\frac{\partial}{\partial\theta}+$$

$$\left[\left(\frac{4}{3H^2R(x)}\right)\left(-E_{21}-2E_{66}+\left(\frac{4}{3H^2}\right)(2H_{66}+H_{12})\right)\right]\frac{\partial^3}{\partial x^2\partial\theta}$$

$$F_{54}=\left[\left(\frac{4\sin\alpha}{3H^2R^2(x)}\right)\left(-2E_{22}-2E_{66}+\left(\frac{4}{3H^2}\right)(H_{22}+H_{66})\right)+\left(\frac{\sin\alpha}{R^2(x)}\right)(C_{22}+C_{66})\right]\frac{\partial}{\partial\theta}$$

$$\left[\left(\frac{4}{3H^2R(x)}\right)\left(-2E_{12}-2E_{66}+\left(\frac{4}{3H^2}\right)(H_{12}+H_{66})\right)+\left(\frac{1}{R(x)}\right)(C_{12}+C_{66})\right]\frac{\partial^2}{\partial x\partial\theta}$$

$$F_{55}=\left[\left(\frac{4\sin\alpha}{3H^2R(x)}\right)\left(-2E_{66}+\left(\frac{4}{3H^2}\right)(H_{66})\right)+\left(\frac{\sin\alpha}{R(x)}\right)(C_{66})\right]\frac{\partial}{\partial x}+$$

$$\left[\left(\frac{4\sin^2\alpha}{3H^2R^2(x)}\right)\left(2E_{66}-\left(\frac{4}{3H^2}\right)(H_{66})\right)-\left(\frac{\sin^2\alpha}{R^2(x)}\right)(C_{66})-A_{44}+\left(\frac{8}{H^2}\right)\left(C_{44}-\left(\frac{2}{H^2}\right)E_{44}\right)\right]+$$

$$\left[\left(\frac{4}{3H^2}\right)\left(-2E_{66}+\left(\frac{4}{3H^2}\right)(H_{66})\right)+(C_{66})\right]\frac{\partial^2}{\partial x^2}+$$

$$\left[\left(\frac{4}{3H^2R^2(x)}\right)\left(-2E_{22}+\left(\frac{4}{3H^2}\right)(H_{22})\right)+\left(\frac{1}{R^2(x)}\right)(C_{22})\right]\frac{\partial^2}{\partial\theta^2}$$

تحليل الانبعاج للقشريات الطباقية تحت حمل ضغط محوري

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الخلاصة:

تحليل الانبعاج للقشريات الطباقية تحت حمل ضغط محوري تم بحثها باستعمال نظرية القشريات ذات درجة عليا ونظرية لوف للقشريات. استخدمت متسلسلات القوة لحل معادلات الحركة التي تم تطويرها للقشريات المخروطية لزوايا مختلفة ، نسبة طول الى نصف قطر ، عدد الطبقات والظروف الحدية. صحة هذه الطريقة تم تعزيزها.