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Optimum Height of Plate Stiffener under Pressure Effect

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Abstract

The economical design of plate loaded by pressure can be obtained by using stiffeners instead of increasing the thickness of plate. The main subject of this work is to obtain the effect of stiffener height on the maximum stress in the plate subjected to pressure load. Different plate-stiffener sets are selected to find the effects of stiffener thickness, plate dimensions and pressure, on the optimum stiffener height. The models under consideration are square plates clamped rigidly from four edges.

Finite Element method is used to analyze 160 different models by using the Finite Element software package ANSYS version 11. Another analysis method based on maximum stress equation is used to analyze 30 models. The graphical comparison of results between two analytical methods is presented by a figure. It is found that the numerical results obtained by Finite Element Analysis converge to theoretical results.

The optimum height of stiffener under above different effects is found. The critical pressure curves for square plate dimension sets are also presented.

Keywords : optimum height , stiffeners , square plate , stress , Finite Element Analysis

Introduction

Typical stiffened sheet structures are metal sheets reinforced by stiffeners and are widely used in airframes, wings of aircraft, ship plating includes deck and side plating and chemical industry structures for their light weight and high strength and stiffness [1]. These applications are need for stiffened structural elements , to avoid using of thick plates that produce high weight for these structures and then produce suitable parts for practice applications. So, stiffened plates nowadays are useful for these applications in modern industry. Stiffeners in stiffened plate make it possible to resist highly directional loads

and provide protection against damage and crack growth under both compressive and tensile loads.

Many researches have been published regarding the stiffened plates and their applications in ships, bridges, tank roofs, vehicles, etc [2].

The main aims of this research is to find the optimum height of rib or stiffener that makes the plate stiff under pressure however the pressure value increased . With this goal, minimum weight of material will be gained, the cost of plate-stiffener system used in assembly of equipment will be decreased and the assembly will be useful for structural applications need for light weights like aircrafts and ships parts.

REVIEW OF LITERATURE

The optimum locations of the ribs for a given set of design constraints is studied by Mohammed M. Hasan [3] by using Finite Element Analysis method, he found that the best design is by using stiffeners on both of the two sides of square plate and it gives better results. The relationship between the deflection of a plate clamped by all its edges subjected to pressure and the height of stiffener was studied by Omri Pedatzur [4]. It is found that the curve which represents this relationship try to be tangent to axes of zero displacement when the height of stiffener is increased, i.e. the value of displacement try to be minimum and its variation will be very small when the height of rib increases.

In general, analytical and exact solutions for plate or shell behavior are desirable because of their ease of use and the insight they provide to the designer. These solutions are generally only applicable for small deflection. Numerical techniques, such as Finite Element Analysis, boundary element analysis, and Finite Difference Analysis, can be more accurate in predicting stresses and deflections, especially for large deflections.

The use of plate theory is appropriate for the analysis of plates or shells; therefore, this work has been achieved by using the Finite Element software package ANSYS with plate bending and shell elements.

Thin sheet stressed in one dimension was discussed by W. Flugge [7]. It is assumed that the plate was under pressure and simply supported from its two edges. An element of length dx have been cut from the plate. It is obtained that the stress generated in the plate without stiffener is as expressed in eq.(1)

$$\sigma = \frac{1}{2}\sqrt[3]{\frac{E.p^2.Wp^2}{3tp^2}} \qquad (1)$$

The stress on plate which have stiffeners will be as eq.(2) :

$$\sigma = \frac{1}{2}\sqrt[3]{\frac{E.p^2.Wp^2}{3tp^2}}.\sqrt[3]{\frac{A_1}{A_1+tp}} \qquad (2)$$

Equations (1) and (2) yield good results if the ratio of the sides length is 1:4 or even 1:3, but when the plate is square they will be inapplicable .

The maximum moment for clamped plate is given by S. Timoshenko [9] as in eq. (3):

$$Mmax = -0.0783\,p.(Wp/2)^2 \qquad (3)$$

The flexural rigidity, Dp, of the plate without stiffener [10] is :

$$Dp = \frac{E.tp^3}{12\left(1-\nu^2\right)} \qquad (4)$$

While the flexural rigidity, Ds, of the combined plate and stiffener is :

$$Ds = \frac{E.Is}{Wp} \qquad (5)$$

Therefore , the total flexural rigidity, D_T, of a plate-stiffener system will be $\left(Dp + Ds\right)$. The maximum stress resulted by a pressure on the plate surface clamped from four edges will be as in eq.(6):

$$\sigma_{xs} = \frac{E.Mmax}{12\left(1-\nu^2\right).D_T}.C_S \qquad (6)$$

limits of plate rigidity

The flexural properties of a plate depend greatly upon its thickness in comparison with other dimensions . Plates may be classified into three groups according to the ratio Wp/tp. These groups are:

1. The first group is presented by thick plates having ratios $Wp/tp \le (8\text{ to}10)$. The analysis of such bodies includes all the components of stresses, strains, and displacements as for solid bodies using the general equations of three-dimensional elasticity.

2. The second group refers to plates with ratios $Wp/tp \ge (80\text{ to}100)$. These plates are referred to membranes and they are devoid of flexural rigidity. Membranes carry the lateral loads by axial tensile forces (and shear forces) acting in the plate middle surface . These forces are called membrane forces; they produce projection on a vertical axis and thus balance a lateral load applied to the plate-

membrane. This work deals with this type of plates.

3.The most extensive group represents an intermediate type of plate, so-called thin plate with $(8\ \text{to}10) \le Wp/tp \le (80\ \text{to}100)$. Depending on the value of the ratio δ/tp the part of flexural and membrane forces here may be different. Therefore, this group, in turn, may also be subdivided into two different classes.

a. Stiff plates: A plate can be classified as a stiff plate if $\delta/tp = 0.2$. Stiff plates are flexurally rigid thin plates. They carry loads two dimensionally, mostly by internal bending and twisting moments and by transverse shear forces. The middle plane deformations and the membrane forces are negligible. In engineering practice , the term plate is understood to mean a stiff plate, unless otherwise specified . A fundamental feature of stiff plates is that the equations of static equilibrium for a plate element may be setup for an original (undeformed) configuration of the plate.

b. Flexible plates: If the plate deflections are beyond a certain level, $\delta/tp \ge 0.3$, then , the lateral deflections will be accompanied by stretching of the middle surface . Such plates are referred to flexible plates . These plates represent a combination of stiff plates and membranes and carry external loads by the combined action of internal moments , shear forces , and membrane (axial) forces . This type of plates are widely used by the aerospace industry. When the magnitude of the maximum deflection is considerably greater than the plate thickness, the membrane action predominates. So, if $\delta/tp > 5$, the flexural stress can be neglected compared with the membrane stress. Consequently, the load-carrying mechanism of such plates becomes of the membrane type, i.e., the stress is uniformly distributed over the plate thickness.

Modeling Properties And Dimensions

Square plate is used in this work. The dimensions of plate is given in **Table 1** and shown in **Fig. 1** and **Fig. 2**.

Table 1 : Dimensions of plates

Wp (m)	Hp (m)	tp (mm)
1	1	2
1	1	4
1	1	6
0.5	0.5	2
0.75	0.75	2
1.25	1.25	2
1.5	1.5	2

Many shapes of stiffeners may be used to strength plates or shells to increase the stiffness of these structures such as flat, L-shape, trapezoidal or other shapes [5].

To achieve this study, one longitudinal flat stiffener is used to strength the plate as shown in **Fig. 3**. The height of stiffener (hs) varies from 10 to 100 mm with a step of 10 mm, i.e. (10, 20, 30, 40, 50, 60, 70, 80, 90, 100)mm. Thickness of stiffener (ts) varies from 2 to 6 mm, i.e. (2, 4, 6)mm. Length of stiffener (Ws) is used to be equal to Wp. dimensions of stiffeners are shown in **Fig. 1** and **Fig. 2**.

Material of both plate and stiffener is stainless steel with E = 196.5 GPa, G = 77.221 GPa, ν = 0.27 and ρ = 7834.6 kg/m^3 .

Finite Element Modeling

Finite element computer programs such as NASTRAN, PATRAN, MARC, CATIA, ANSYS are used in large numbers of researches that deal with analysis of stiffened plates. Finite element analysis consists of four steps: creating the geometry of the model, generating a mesh for the solid model (i.e. dividing the model into elements), applying appropriate boundary and loading conditions, and solution. When the meshing of the model and assigning of the material properties are completed, the appropriate load and boundary conditions are applied at the element nodes. Once stiffness equations describing individual elements are constructed, the global stiffness matrix is assembled. The unknown displacement vector to be solved can be symbolically written as in eq.(7) [6]:

$$\{u\} = [K]^{1}.\{F\} \quad (7)$$

where {u} is the vector of nodal unknown displacements that the program computes, {F} is the vector of load applied, and [K] is the global stiffness matrix (generated automatically by ANSYS, based on the geometry of model and properties of material). Finite element analysis with geometric and material nonlinearities is carried out herein using the software ANSYS version 11.

Materials having the same properties in one plane (e.g., y, z) and different properties in the direction perpendicular to the plane (e.g., x direction) are called isotropic. An isotropic material may become anisotropic due to cold working and forging. A material whose properties vary in three orthogonal directions is called orthotropic [7] . This work based on the isotropic plate theory. This method analyzes the plate stiffener system as a plate of equivalent uniform thickness.

The accuracy of the finite element analysis results highly depends on choosing the appropriate elements to predict the actual behavior of the structure . Both plate and stiffener are very thin, with high width to thickness ratio. The elastic shells type 63 and type 93 elements, are the most advanced shell elements in the ANSYS element library, and they were determined to be the best choice to model both the stiffener and the plate. These elements are suitable for analyzing thin to moderately thick shell structures and they have orthotropic properties . In this research , shell type 93 element is used to represent the plate elements while shell type 63 element represents the stiffener elements . Shell type 93 element is an eight node element with six degrees of freedom at each node: translations in the x, y and z and rotations about the nodal x, y, and z-axes. The element has plasticity, large deflection, and large strain capabilities, as shown in **Fig. 4**.

Shell type 63 element is a four node element with six degrees of freedom at each node : translations in the x, y and z directions and rotations about the x, y, and z axes, as shown in **Fig. 5**. This element has bending and membrane capabilities.

Meshing, Loading, Boundary Conditons Of Models

The geometry of the plate and stiffener is modeled as shown in **Fig. 3**.

After creating the solid geometry, the plate and stiffener areas were meshed with shell 93 and shell 63 elements respectively. In this work, the number of element edge length for plate is used to be twice times as for stiffener, i.e. the number of element edge length for plate is 20 element in the x direction and 20 element in the y direction. While 10 element in the x direction and 10 element in the y direction are the numbers of element edge length for stiffener. The accuracy of numerical results depends on the number of the elements of geometry mesh.

All sets are loaded by pressure, as shown in **Fig. 1**.

One case of design constraint is used in this work. **Fig. 1** and **Fig. 2** show this case where all of the edges of square plate are fixed.

Results And Discussions

The optimum height of stiffener is based on a criteria of allowable stress for the plate-stiffener system under pressure load. This system is studied under static pressure load.

The plate-stiffener system clamped along its four edges, subjected to pressure, may be affected by many variables such as plate dimensions, stiffener thickness, stiffener height, pressure. The effect of increasing stiffener height for three different plate dimensions on the stress produced in the plate is obtained as shown in **Fig. 6**.

It is a fact, that the stiffener is more effective in a square plate have larger dimensions (length and width). It is found that increasing in stiffener height for three square plates dimensions selected reduces the stress generated in plate-stiffener system, until it reaches a value, however the stiffener height increases, the stress varies in a very small value and the curves shown in **Fig. 6** seem to be like a line.

This study is concerned on minimizing the weight of plate-stiffener system by reducing the height of stiffener and using only one stiffener.

A comparison is made between three different thickness of plates stiffened by ribs, as shown in **Fig. 7**. The stiffener in the thicker plate has a little effect on the stresses resulted in plate-stiffener system, while the effect of stiffener is clear on the stresses in the thinner plate.

It is found that the stress resulted in the thinner plate stiffened by a rib reach to the maximum value when the stiffener have minimum height. Thicker plate is more stiff than thin plate, therefore the stiffener is more effective in thin plate than in thick plate. It is found that the stresses resulted in thin plate stiffened by a rib are the maximum when the height of stiffener is minimum, as shown in **Fig. 7**. The minimum values of stress resulted in plate-stiffener system for the three selected plates are found at certain values of stiffener height. It can be shown that, when, the height of stiffener increases above certain values, there is no significant effect on stresses, as shown in **Fig. 7**.

Another comparison is related to the thickness of stiffeners. It is made among three different thickness of stiffeners, as shown in **Fig. 8**.

It is clear that the thicker stiffener has a significant effect on the stresses resulted in the plate-stiffener system, than the thinner stiffener. It is obtained that the three curves which represent the relation between the height of stiffeners and the maximum stresses resulted in the plate-stiffener system start at maximum values of stresses at minimum height of stiffeners. Then, these curves tend to fall down until they converge one from another and continue as straight lines at a certain value of stress. However as the height of stiffeners are increased, the stresses are still converge from this certain value.

In this work, the critical pressure is defined as a pressure which results operating stress equal to the allowable stress of plate-stiffener system material at selected height of a stiffener. While the critical curve is defined as a curve pass through the critical pressures at all of the selected heights of a stiffener.

The area under the critical curve should be defined as a stable area. At stable area, the plate-stiffener system can be stable, stiff and carry the pressure applied on it without failure. While the area above the critical curve should be defined as unstable area. In which, the system can not be stable nor stiff under the pressure applied. The previous definitions is shown in **Fig. 9**.

The behavior of the curves of the results obtained by theoretical analyses method are rather similar to the curves of the results obtained by finite element analyses method with some differences, as shown in **Fig. 1٠**.

The Finite element method analyses predicted the effect of stiffener in the two directions x and y for whole system with the same accuracy, while the calculations by theoretical method is applied for plate only. For this reason, the difference between the curves of two analysis methods is clear, as shown in **Fig. 10**.

Conclusions

From results of this work, it is obtained that there is optimum height for a flat stiffener in a plate-stiffener system loaded by a pressure and clamped from its four edges. It is found that, however the dimensions of square plate, thickness of stiffener and pressure vary in their values, the optimum height of stiffener is still about a range between 40mm and 50mm. Any value above this range will not be useful and not economic, because it will not reduce the maximum stress resulted in the plate-stiffener system. The obtaining of optimum height of stiffener reduces the weight of stiffener by about 50%, and this is useful to get light weight and low cost of plate-stiffener system.

Critical curves for square plates that have range of dimensions from (500x500)mm^2 to (1500x1500)mm^2 step (250x250)mm^2 are obtained, as shown in **Fig. 11**. This figure is useful for a designer to choose the suitable operating pressure, as a quick method without using calculations.

Symbols

A_1	Area of cross section of one stiffener, it is equal $A_1 = A/d$ (mm^2).
A	Area of cross section of one stiffener, it is equal to (ts.Ws), (mm^2).
Ap	Area of cross section of plate, it is equal to (0.5 tp.Wp), (mm^2).
c	Natural axis of the combined plate and stiffener, it is equal to: $\left(\dfrac{\frac{1}{2}A.Ws - \frac{1}{2}Ap.tp}{A + Ap}\right)$, (mm)
C_S	Distance from the surface of stiffener to the natural axis of plate- stiffener system, (mm).
d	Distance between two stiffeners, (mm).
Dp	Flexural rigidity of the plate without stiffener, (N/mm).
ds	Distance from mid plane of stiffener to the natural axis of the combined plate and stiffener, it is equal to $(0.5Ws - c)$, (mm).
Ds	Flexural rigidity of the combined plate and stiffener, (N/mm).
D_T	Total flexural rigidity of a plate-stiffener system, (N/mm).
E	Young Modulus of Elasticity, (GPa).
{F}	Vector of load applied.
G	Shear Modulus of Rigidity, (GPa).
hs	Height of the stiffener, (mm).
Hp	Length of the plate, (mm).
I, J, K, L, M, N, O, P	Nodes of element.
Is	Moment of inertia of a stiffener, taken with respect to the middle axis of the cross section of the plate, it is equal to: $\left(\dfrac{Ws^3.ts}{12} + A.ds\right)$ (mm^4).
{K}	Global stiffness matrix (generated automatically by ANSYS, based on the geometry of model and properties of material.
Mmax	Maximum moment on plate under pressure clamped by its four edges, (N.mm).
Mx	Moment about x-coordinate, (N.mm).

Mxy	Twisting moment in xy plane, (N.mm).
My	Moment about y-coordinate, (N.mm).
Nz	Force normal to element surface in z-coordinate, (N).
P	Pressure applied on the plate, (N/mm^2)
tp	Thickness of a plate, (mm).
ts	Thickness of a stiffener, (mm).
Tx	Element force in x-coordinate, (N).
Txy	Element force in xy plane, (N).
Ty	Element force in y-coordinate, (N).
{u}	Vector of nodal unknown displacements that the program computes.
Wp	Width of a plate, (mm).
Ws	Length of the stiffener, (mm).
ν	Poisson's ratio.
ρ	Mass density, (kg/m^3).
δ	Maximum deflection of the plate, (mm).
σ	Direct stress along the plate, (N/mm^2).
σ_{xs}	Maximum stress resulted by a pressure on the plate surface clamped from four edges, (N/mm^2).

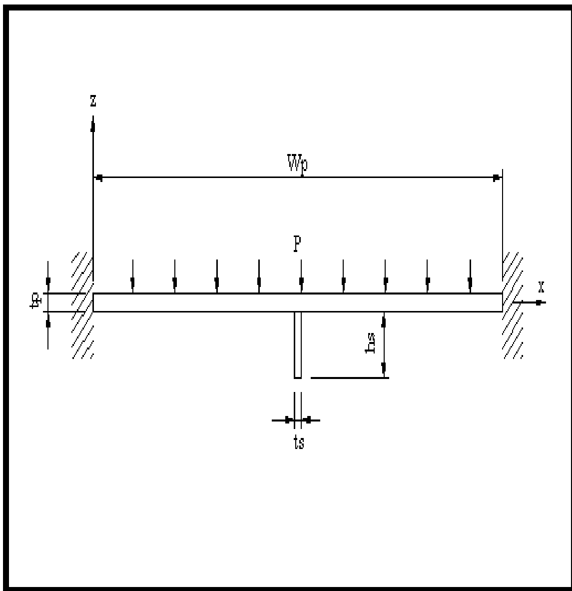

Fig. 1: Side view of the model

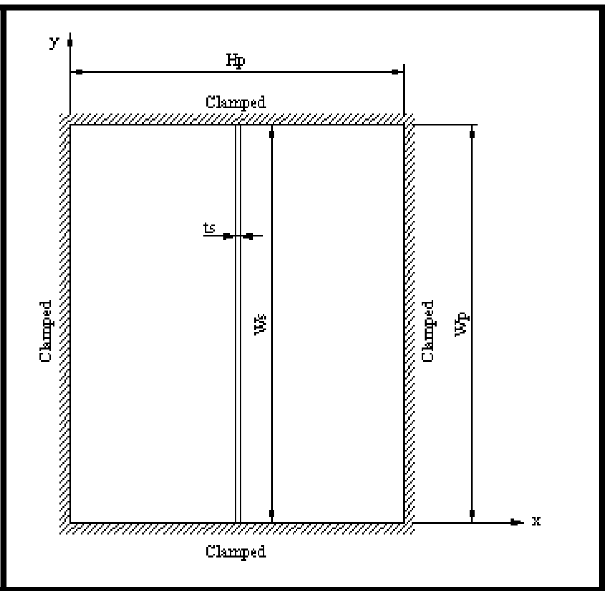

Fig. 2: Bottom view of the model

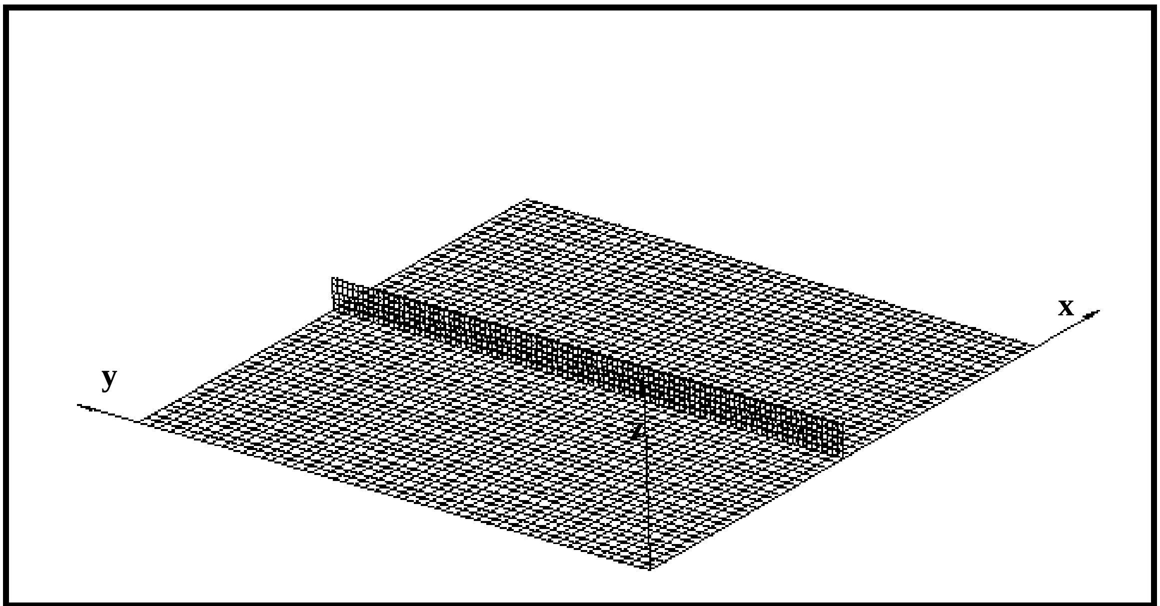

Fig. 3: Meshed plate-stiffener system in ANSYS program

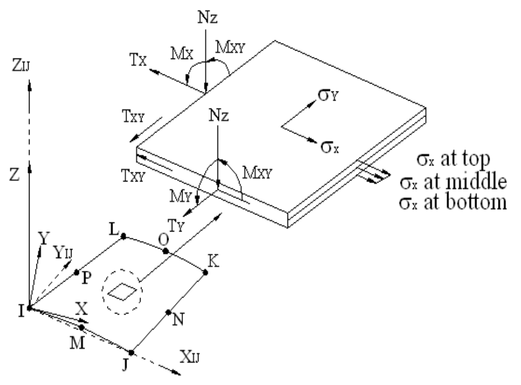

Fig. 4: Shell type 93 element

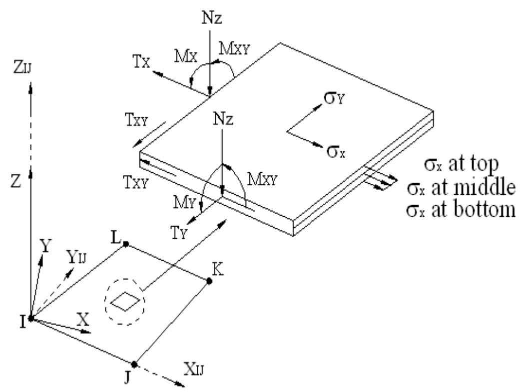

Fig. 5: Shell type 63 element

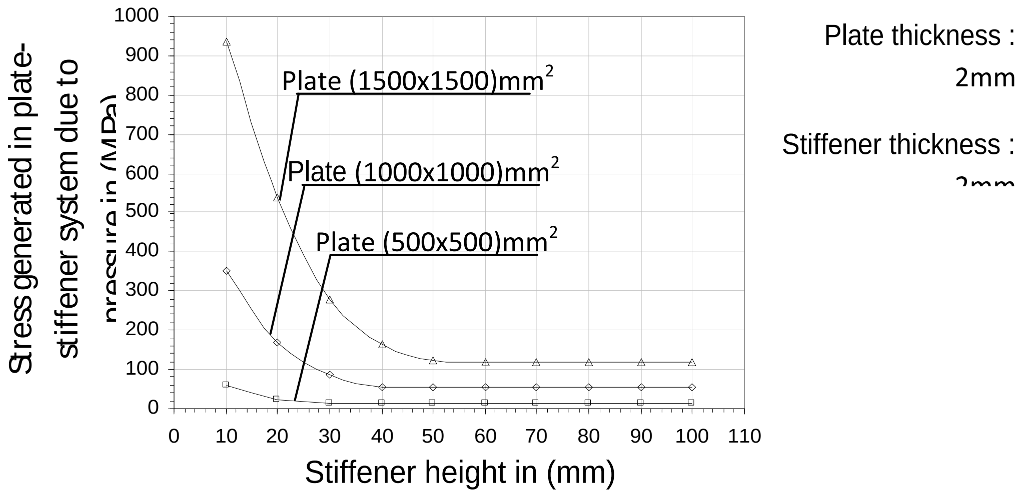

Fig. 6 : Effect of plate dimensions on stress generated in a square plate having one stiffener and clamped along its 4 edges loaded by pressure

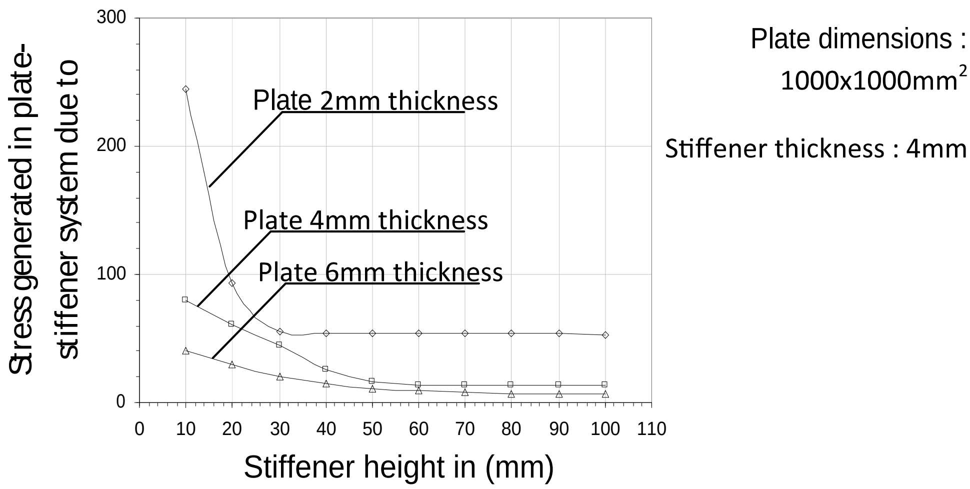

Fig. 7: Effect of plate thickness on stress generated in a square plate having one stiffener and clamped along its 4 edges loaded by pressure

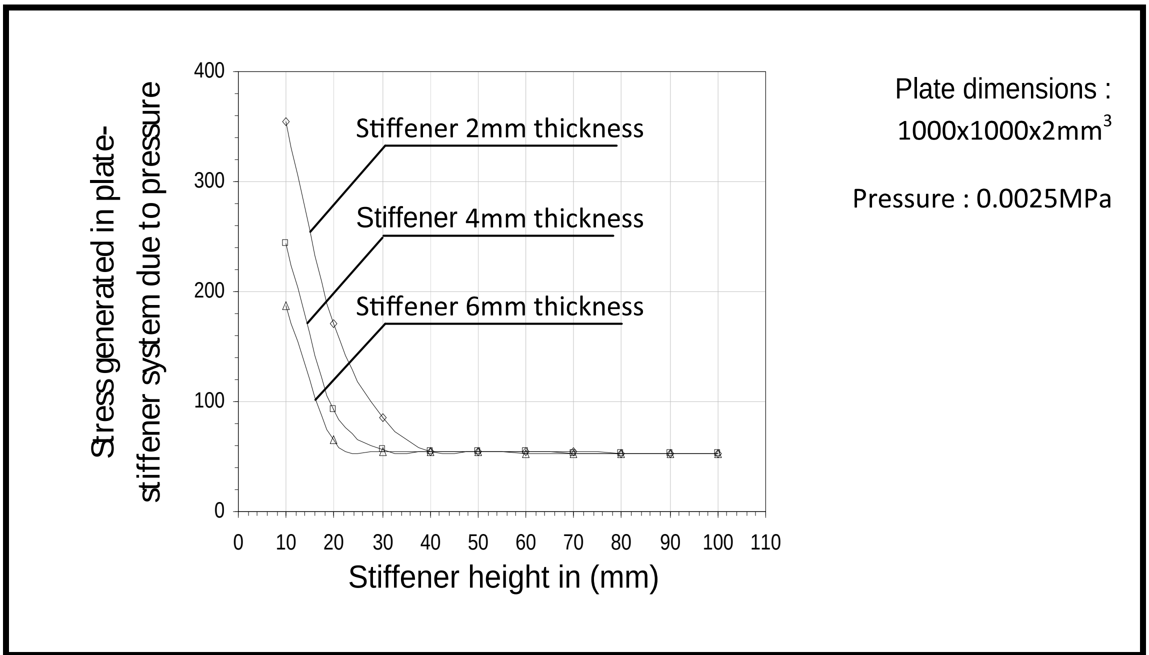

Fig. 8: Effect of stiffener thickness on stress generated in a square plate having one stiffener and clamped along its 4 edges loaded by pressure

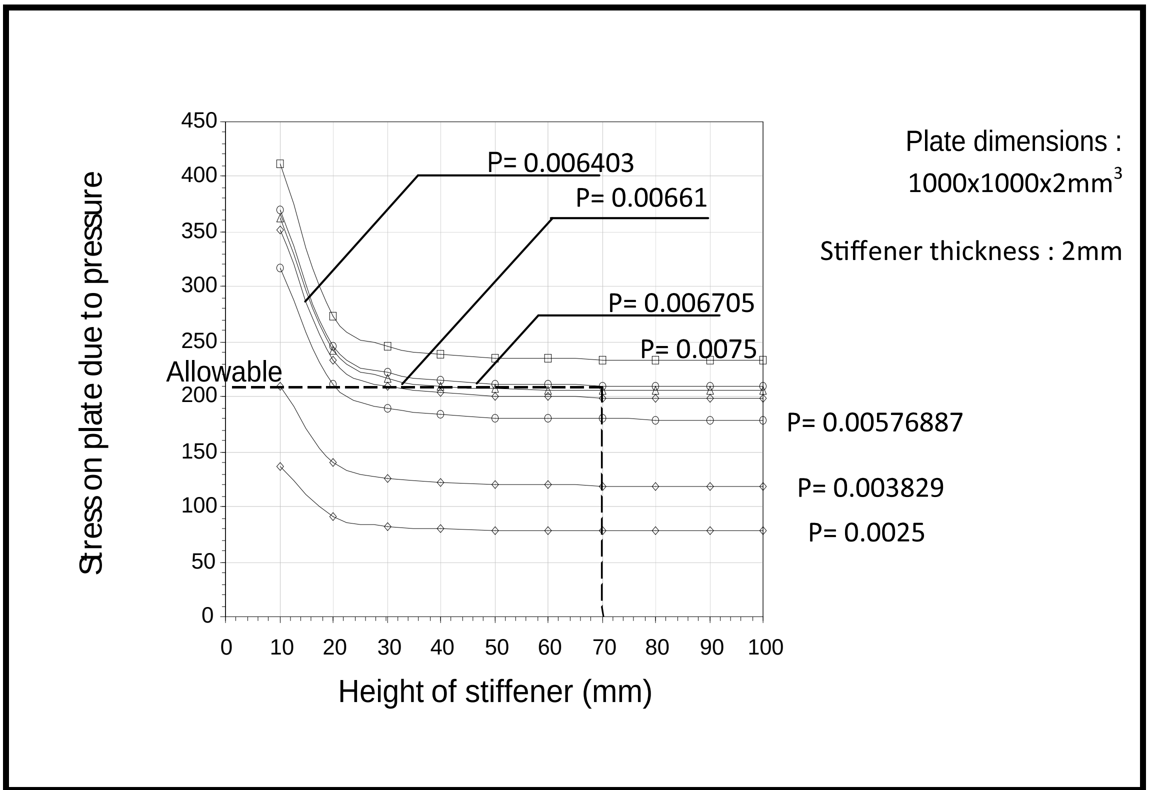

Fig. 9 : Square plates clamped along its 4 edges stiffened by one stiffener loaded by uniform pressure

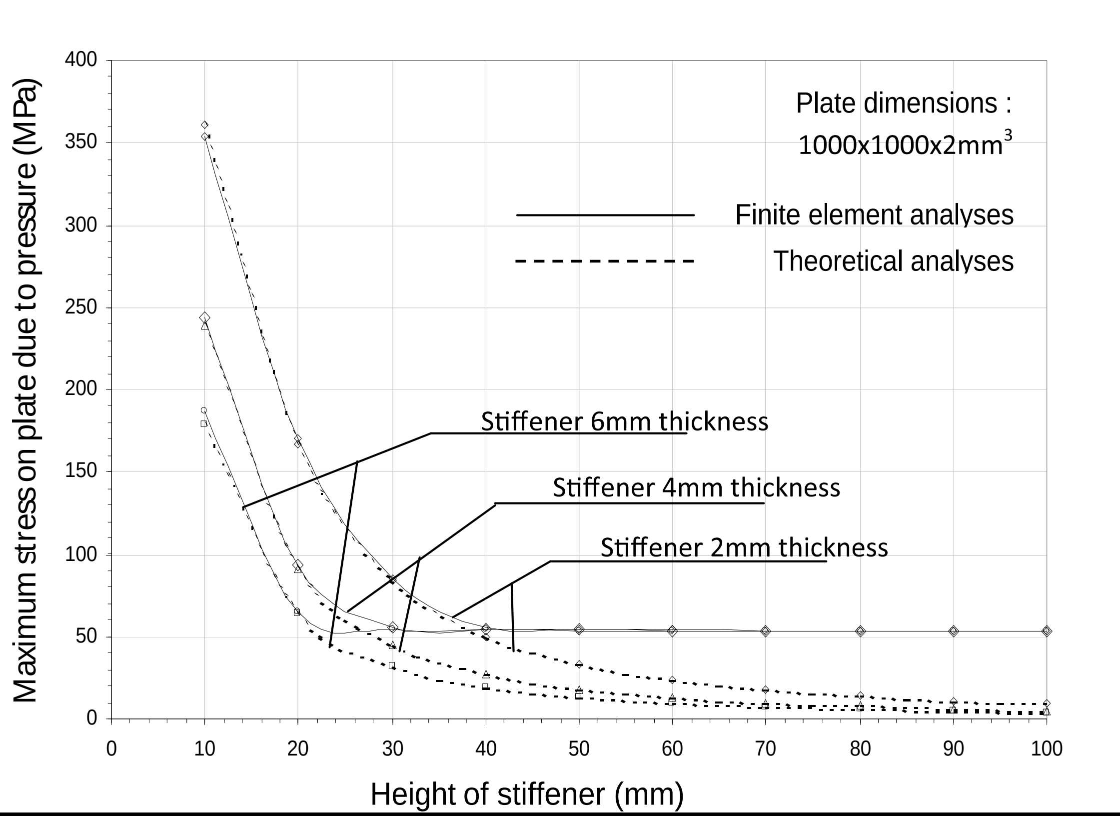

Fig. 10 : Comparison between finite element and theoretical analyses methods to compute maximum stress in stiffened square plate clamped along its 4 edges subjected to uniform pressure

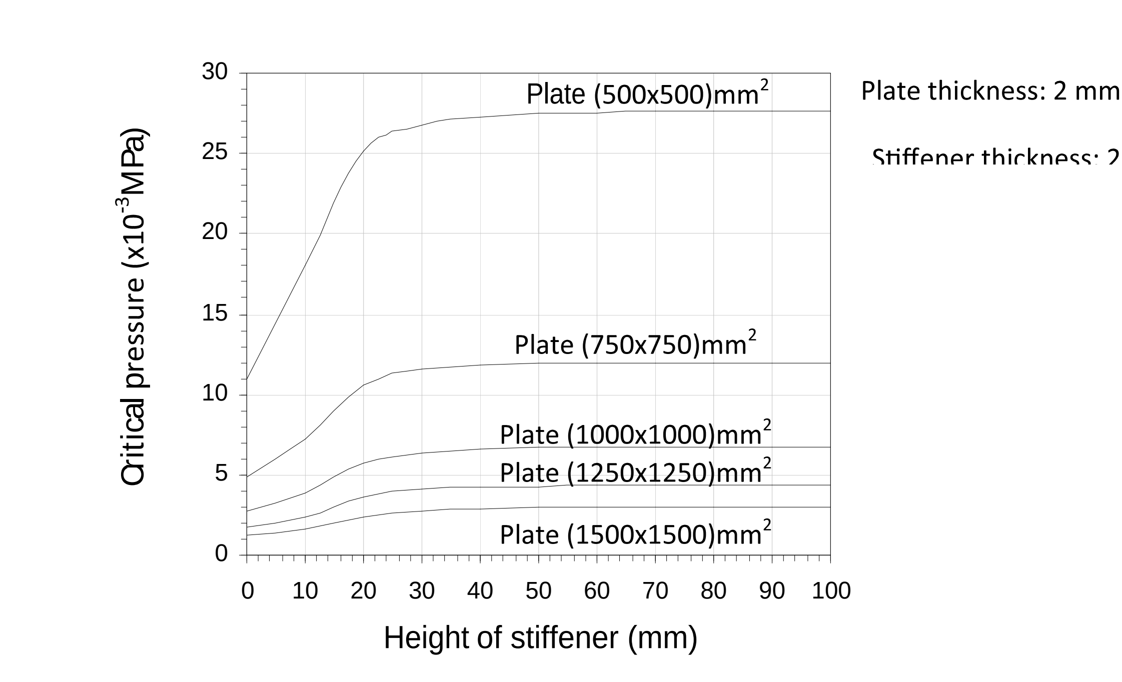

Fig. 11 : Critical curves for square plate clamped along 4 edges

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الإرتفاع الأمثل للتقوية في الصفائح تحت تأثير الضغط

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ملخص البحث

يمكن الحصول على التصميم الإقتصادي للصفيحة المعرضة الى ضغط بإستخدام أضلاع تقوية بدلاً من زيادة سمك تلك الصفيحة. إن الموضوع الرئيسي في هذا العمل هو إيجاد تأثير إرتفاع ضلع التقوية على أكبر إجهاد يتولد في الصفيحة المعرضة الى ضغط. لقد أختيرت مجاميع مختلفة من الصفائح المقواة وذلك لإيجاد تأثير سمك التقوية، أبعاد الصفيحة والضغط المسلط، على الإرتفاع الأمثل للتقوية. إن النماذج المأخوذة بنظر الإعتبار هي عبارة عن صفائح مربعة مسندة بإحكام من حافاتها الأربعة.

أستخدمت طريقة العناصر المحددة لتحليل ١٦٠ نموذج مختلف وذلك بالإستعانة ببرنامج العناصر المحددة الـ (ANSYS) الإصدار ١١. إن الطريقة الأخرى للتحليل التي تعتمد على معادلة أكبر إجهاد أستخدمت لتحليل ٣٠ نموذج. إن مقارنة رسومية للنتائج بين طريقتي التحليل تم تمثيلها بشكل. وقد وجد أن النتائج العددية المستحصلة بواسطة التحليل بالعناصر المحددة تقترب من النتائج النظرية.

لقد تم إيجاد الإرتفاع الأمثل لضلع التقوية بوجود التأثيرات المختلفة المذكورة سابقاً. وتم تمثيل منحنيات الضغط الحرج الخاصة بمجاميع الصفائح المربعة مختلفة القياسات