# Design of a two-Degree-of-Freedom Controller for a Magnetic Levitation System Based on LQG Technique

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## Abstract:

A new control design procedure has been proposed in this paper based on the *LQG* control design. A two degree of freedom controller with integral action is obtained and tested on the magnetic levitation system, which is a good test-bed for control design because of its nonlinearity and unstability with practical uses in high-speed transportation and magnetic bearings. Simulations are performed under MATLAB environment and included to highlight that the proposed controller accurately achieves position tracking for different kinds of reference inputs.

## Keywords:

LQG, Kalman filter, Magnetic levitation, Integral action, two-degree-of freedom.

## I. Background And Motivation

Magnetic levitation systems have practical importance in many engineering systems suchas in high-speed maglev passenger trains, frictionless bearings, levitation of wind tunnelmodels, vibration isolation of sensitive machinery, levitation of molten metal in inductionfurnaces, and levitation of metal slabs during manufacturing. The magnetic levitation systemscan be classified as attractive systems or repulsive systems based on the source of levitationforces. These kinds of systems are usually open-loop unstable and are described byhighly nonlinear differential equations which present additional difficulties in controllingthese systems. Therefore, it is an important task to construct high-performance feedbackcontrollers for regulating the position of the levitated object [1].

In recent years, a lot of works have been reported in the literature for controlling magnetic levitation systems. The feedback linearization technique has been used to design control laws for magnetic levitation systems [2]. The input-output, input-state, and exact linearization techniques have been used to develop nonlinear controllers [3,4]. Other types of nonlinear controllers based on nonlinear methods have been reported in the literatures [5,6,7]. Robust linear controller methods such as H∞ optimal control, μ-synthesis, and Q-parameterization have also been applied to control magnetic levitation system [8,9,10,11]. Due to the features of the instability and nonlinearities of the magnetic suspension system, authors in [12] presents a magnetic levitation ball control system based on TMS320F2812, which is a high performance Digital Signal Controller currently in use in control engineering field.

Various control schemes based on neural networks (NN) techniques have been proposed for magnetic levitation system in the literature, in [13] a feedback error learning together with a PID has been used a hybrid control to guarantee stability of control approach. A method of simple adaptive control using neural networks with offset error reduction for an SISO magnetic levitation system is introduced in [14]. In this method the role of neural networks is to compensate for constructing a linearized model so as to minimize the output error caused by nonlinearities in the magnetic levitation system. While the work in [15] incorporate a NN in model reference adaptive control (MRAC) to overcome the problem. The control input is given by the sum of the output of the adaptive controller and the output of the NN. The NN is used to compensate the nonlinearity of the plant that is not taken into consideration in the conventional MRAC. Authors in [16] propose a hybrid controller using a recurrent neural network (RNN) to control a levitated object in a magnetic levitation system, to ensure the convergence of the RNN, the adaptation law of the RNN is modified by using a projection algorithm, and [17] presents an adaptive neural fuzzy network (ANFN) controller based on a modified differential evolution (MODE) for solving control problems. During the last two decades, sliding mode control (SMC) have received significant interest and have become well- established research areas with great potential for practical applications. Research in [18,19] used the magnetic force model of [20] and proposed sliding mode controllers (SMC) for magnetic levitation systems. Combination between SMC and intelligent design have been proposed in

[21,22,23]. The fuzzy control technique is also used in magnetic levitation system as described in [24], while authors in [25] presented an optimum approach for designing controller for magnetic levitation system based on genetic algorithm (GA). Combinations between different intelligent designed methods like neural-fuzzy and their application to magnetic levitation system are investigated in [ 26, 27].

This paper is organized as follows: Section 2 describes the linear quadratic Gaussian (LQG), section III introduces the mathematical modeling of the magnetic levitation system, section IV is devoted to the design of the 2DOF controller procedure. Simulations and results are presented in section V. Finally, the conclusions are given in section VI.

## II. Linear Quadratic Gaussian (Lqg) Control Design

In traditional LQG control, it is assumed that the plant dynamics are linear and known, and that the measurement noise inputs and disturbance signals (process noise) are stochastic with known statistical properties. That is, we have a plant model [28]:

$$\dot{x} = Ax + Bu + \omega_d \qquad (1)$$

$$y = Cx + Du + \omega_n \qquad (2)$$

Where for simplicity we set $D = 0$. $\omega_d$ and $\omega_n$ are the disturbance (process noise) and measurement noise respectively, which are usually assumed to be uncorrelated zero-mean Gaussian stochastic processes with constant power spectral density matrices $W$ and $V$ respectively. That is, $\omega_d$ and $\omega_n$ are white noise processes [28].

The *LQG* problem is to find the optimal control $u(t)$ which minimizes [28,29],

$$J = E\left\{ \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[ x^T Q\ x + u^T R\ u \right] du \right\} \qquad (3)$$

Where $E\{ \}$ is the expectation operator, $Q$ and $R$ are appropriately chosen constant weighting matrices (design parameters) such that $Q = Q^T > 0$ and $R = R^T > 0$. The name *LQG* arises from the use of a Linear model, an integral Quadratic cost function, and Gaussian white noise

processes to model disturbance signals and noise [28].

The solution to the *LQG* problem, known as the separation theorem or certainty equivalence principle, is surprisingly simple and elegant. It consists of first determining the optimal controller for a deterministic *linear Quadratic Regulator* (*LQR*) problem: namely, the above *LQG* problem without $\omega_d$ and $\omega_n$. It happens that the solution to this problem can be written in terms of the simple state feedback law [28,29],

$$u(t) = -K_r x(t) \qquad (4)$$

Where $K_r$ is a constant matrix which is easy to compute and is clearly independent of $W$ and $V$, the statistical properties of the plant noise. Note that eq(4) requires that $x$ is measured and available for feedback, which is not generally the case. This difficulty is overcome by the next step, where we find an optimal estimate $\hat{x}$ of the state $x$, so that $E\left\{ [x - \hat{x}]^T [x - \hat{x}] \right\}$ is minimized. The optimal state estimate is given by a Kalman filter and is independent of $Q$ and $R$. the required solution to the *LQG* problem is then found by replacing $x$ by $\hat{x}$, to give $u(t) = -K_r \hat{x}(t)$. We therefore see that the *LQG* problem and its solution can be separated into two distinct parts, as illustrated in the figure 1 [28].
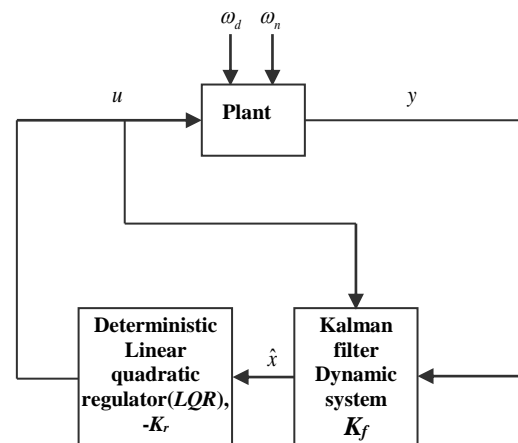
Figure 1: The separation theorem.

The *LQR* problem, where all the states are known, is the deterministic initial value problem: given the system $\dot{x} = Ax + Bu$ with a nonzero initial state $x(0)$, find the input signal $u(t)$ which takes the system to the zero state $(x = 0)$ in an optimal manner, i.e by minimizing the deterministic cost [28],

$$J = \int_0^\infty (x(t)^T Q \; x(t) + u(t)^T R \; u(t)) \, du \qquad (5)$$

The optimal solution (for any initial state) is $u(t) = -K_r x(t)$, where

$$K_r = R^{-1} B^T X \qquad (6)$$

and $X = X^T \geq 0$ is the unique positive semi-definite solution of the algebraic riccati equation,

$$A^T X + X \, A - X \, B \, R^{-1} B^T X + Q = 0 \qquad (7)$$

The Kalman filter has the structure of an ordinary state estimator or observer, as shown in figure 2 below, with [30]:

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f (y - C\hat{x}) \qquad (8)$$

The optimal choice of $K_f$, which minimizes $E\left\{ [x - \hat{x}]^T [x - \hat{x}] \right\}$, is given by [30],

$$K_f = YC^T V^{-1} \qquad (9)$$

Where $Y = Y^T \geq 0$ is the unique positive semi-definite solution of the algebraic Riccati equation [30],

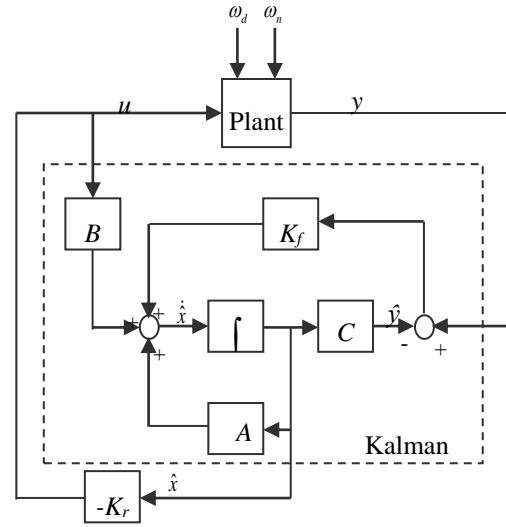$$Y A^T + A Y - Y C^T V^{-1} C Y + W = 0 \qquad (10)$$



Figure 2: the *LQG* controller and noisy plant.

The *LQG* control problem is to minimize *J* in eq( 3). The structure of the *LQG* controller is illustrated in figure 2; but it is not easy to see where to position the reference input *r*, and how the integral action may be included, if desired. one strategy in included in figure 3. Here the control error *r-y* is integrated and the regulator $K_r$ is designed for the plant augmented with the integrator states [28,29].
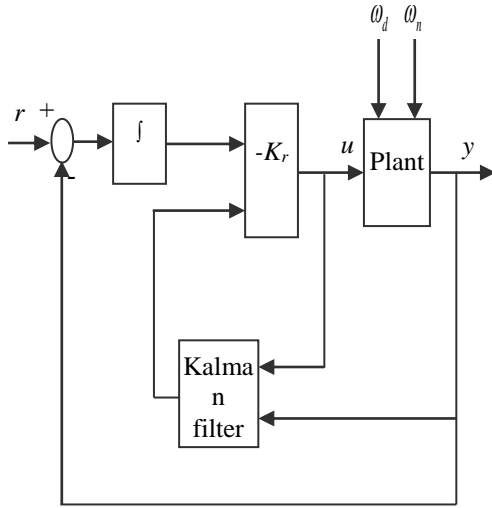
Figure 3: 2-DOF *LQG* Controller with integral action and reference input.

The state-space representation of the augmented plant( original plant with integrator) is given by [28]:

$$A_{aug} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, B_{aug} = \begin{bmatrix} B \\ -D \end{bmatrix}, \qquad (11)$$

$$C_{aug} = [C], \qquad D_{aug} = [D]$$

While the transfer function of the 2-DOF *LQG* controller from [*ry*] to *u* (i.e assuming positive feedback), is easily shown to be given by [28,29]:

$$K_{2-DOF}(s) = \begin{bmatrix} A_C & \vdots & B_C \\ \cdots & & \cdots \\ C_C & \vdots & D_C \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \vdots & I & -I \\ -bK_{ri} & a-bK_{rp}-K_eC & \vdots & 0 & K_f \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \cdots & \cdots \\ -K_{ri} & -K_{rp} & \vdots & 0 & 0 \end{bmatrix} \quad (12)$$

Where $K_{rp}$ and $K_{ri}$ are the gains of the state feedback *LQR* of the integrator and plant states respectively (*i.e.* $-K_r = [-K_{ri} \quad -K_{rp}]$ ) [28,29].

Its order is higher by one than the plant's order. Note that the optimal gain matrices $K_f$ and $K_r$ exist,

and the LQG-controlled system is internally stable provided that the system with state-space realizations ( $A, B, Q^{1/2}$ ) and ( $A, W^{1/2}, C$ ) are stabilizable and detectable [28].

## III. Model of The Magnetic Levitation System

In this section, we will present the model of a magnetically suspended ball system,a schematic of which is shown in Fig. 4. The current passing through the wirewound around the armature creates a magnetic force, which attracts the steel balland counter balances the force due to gravity [31].


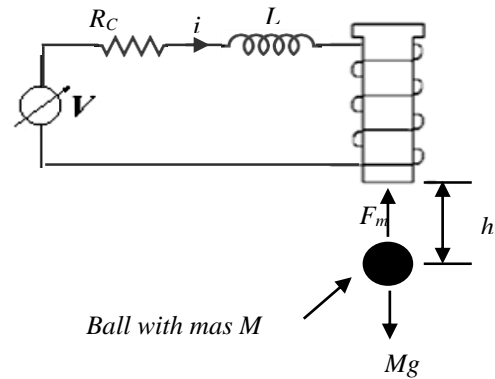
Figure 4: Magnetic Levitation system

The magnetic force is directly proportional to the square of the current and inverselyproportional to the distance between the ball and the armature. The force balancecan be written as [32,33]:

$$M \frac{d^2h}{dt^2} = Mg - F_m = Mg - \frac{K_m i^2}{h^2} \quad (13)$$

where $K_m$ is the proportionality constant. The voltage balance in the circuit can bewritten as [31,32,33]:

$$V = L\frac{di}{dt} + R_C i \qquad (14)$$

Suppose that the current *i* is such that the ball is stationary at a chosen distance $h_s$ .We would like to derive a linear model that relates a deviation in *h* to a deviationin *i*. Let the force balance corresponding to the stationary point be modelled as [32,33,34]:

$$M \frac{d^2h_s}{dt^2} = Mg - \frac{K_m i_s^2}{h_s^2} = 0 \quad (15)$$

Subtracting Eq. (15) from Eq. (13), we obtain [32,34]:

$$M \frac{d^2 \Delta h}{dt^2} = -K_m \left[ \frac{i^2}{h^2} - \frac{i_s^2}{h_s^2} \right] \quad (16)$$

After performing linearization to the nonlinear system (eq(16)) and letting $x_1 \overset{\Delta}{=} \Delta h$ , $x_2 \overset{\Delta}{=} \Delta \dot{h}$ , $x_3 \overset{\Delta}{=} \Delta i$ , $u \overset{\Delta}{=} \Delta V$ , the linearized state-space model of the magnetic levitation system is given by [31,32,33,34]:

$$\dot{x} = Ax + Bu = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 2\frac{K_m}{M}\frac{i_s^2}{h_s^3} & 0 & -2\frac{K_m}{M}\frac{i_s}{h_s^2} \\ 0 & 0 & -\frac{R_C}{L} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u \quad (17)$$

$$y = C x + D u = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

Typical values of the system parameters are given in table 1 [32].

**Table 1: Description of the Magnetic Levitation system Parameters.**

| Parameter | Description | Value | Unit |
|-----------|-------------|-------|------|
| $M$ | Mass of the ball | 0.05 | kg |
| $L$ | Inductance | 0.01 | H |
| $R_c$ | Resistance | 1 | ohm |
| $K_m$ | Proportionalilty costant | 0.0001 | - |
| $g$ | Acceleration of the gravity | 9.81 | m/s² |
| $h_s$ | Distance | 0.01 | m |
| $i_s$ | Current at stationary | 0.7 | A |

The current corresponding to the stationary point is obtained from eq(15) as [32]:

$$i_s^2 = \frac{Mgh_s^2}{K_m} = \frac{0.05 * 9.81 * (0.01)^2}{0.0001} \Rightarrow i_s = 0.7A$$

In other words, we have to apply an input voltage of $V = i_s * R_C = 0.7$ volt (neglecting the voltage drop on the inductor) to maintain the ball at stationary point ($h_s = 0.01$ meter).With these typical values given in table 1, one can arrive at the state-space equation, given by eq(17) with state-space matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1960 & 0 & -28 \\ 0 & 0 & -100 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix} \quad (18)$$

The magnetic levitation system is open-loop unstable with poles (eignvalues of $A$) given as:44.2719, -44.2719, -100. As can be seen one of the poles lie in the right-half plane. This results in a time response due to 0.02 step change of the reference input shown in figure 5.
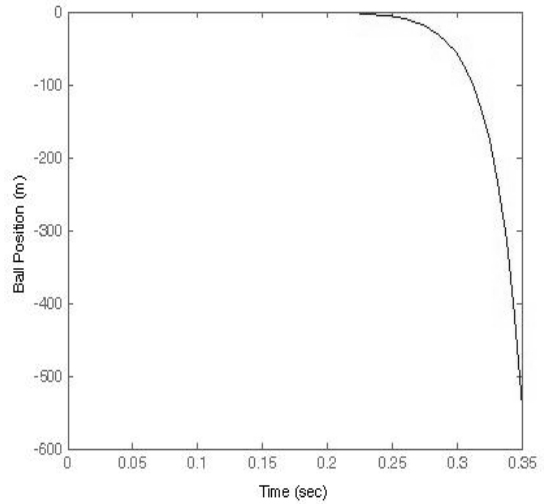


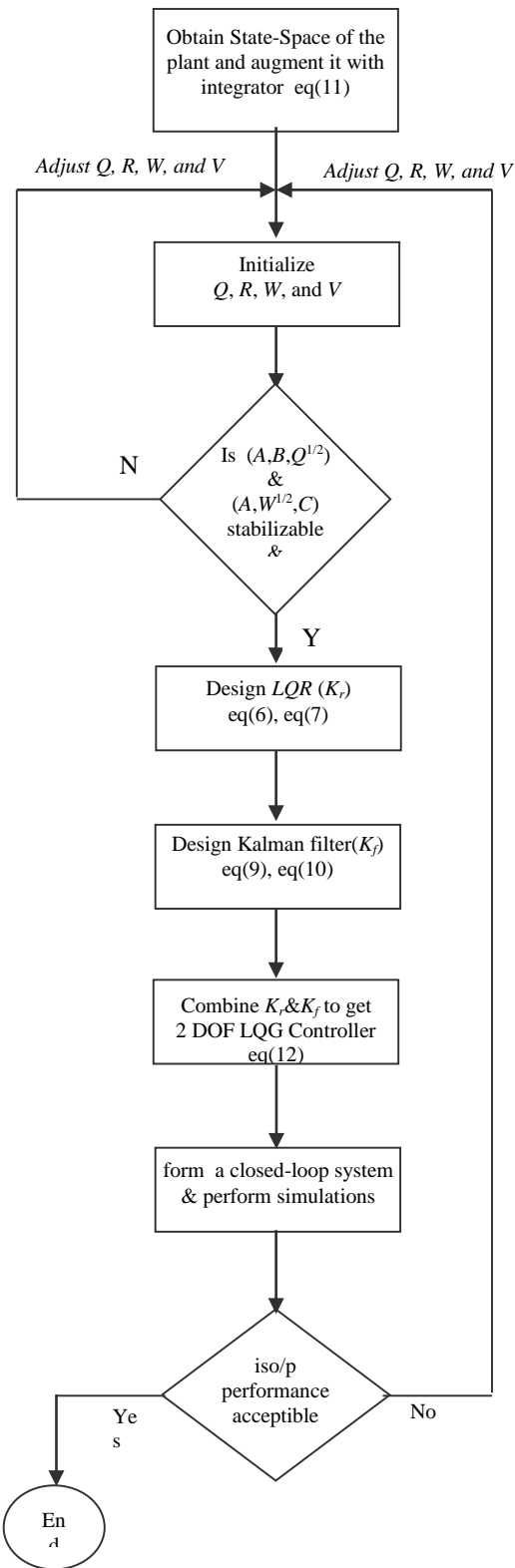Figure 5: Ball position of the magnetic levitation system due to step change of 0.02 meter in the reference input.

## IV. 2-DOF LQG CONTROLLER DESIGN PROCEDURE

The output response of the ball position of the magnetic levitation system is clearly unstable as shown in figure 5, it is growing without bound and indeed it needs a controller to stabilize it and produce an acceptable output performance. Figure 6 is a flowchart that shows the various stages for the 2 DOF LQG controller design.

## V. SIMULATIONS AND RESULTS

Figure 7 is the closed-loop control system used to perform the simulations of the proposed 2DOF LQG controller for magnetic levitation system. To account for negative sign of the $LQR$ gain (*i.e.* $-K_r$), a positive feedback is used in the simulations. The weighting matrix $Q$ is chosen such that only the integrated state $y\text{-}r$ is weighted, while the Kalman filter is setup such that the integrated states are not estimated.
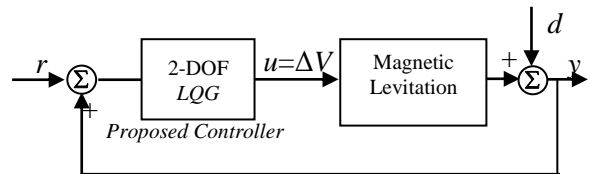
Figure 7: Closed-loop control system of the magnetic levitation system.

The weighting matrix W is chosen so that the process noise directly affects the states, with these assumptions the values of the weighting matrices that are used in this simulations are:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 250 \end{bmatrix} , \quad R = \begin{bmatrix} 10^{-3} \end{bmatrix} ,$$

$$W = \begin{bmatrix} 500 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 500 \end{bmatrix} , \quad V = \begin{bmatrix} 10^{-3} \end{bmatrix}$$

Figure 6: Flowchart of a 2 DOF LQG controller design procedure.

and the 2 DOF LQG controller from $[r y]$ to $u$ (eq(12)) in transfer function form is obtained by running the simulation program through MATLAB environment and found to be:

$$K_{2-DOF}(s) = \begin{bmatrix} \dfrac{-500\,s^3 - 4.046*10^5 s^2 - 3.669*10^7 s - 1.047*10^8}{s^4 + 907.7\,s^3 + 1.55*10^5 s^2 + 1.104*10^7 s} \\[4mm] \dfrac{1.86*10^5 s^3 + 2.692*10^7 s^2 + 8.294*10^8 s + 1.047*10^8}{s^4 + 907.7\,s^3 + 1.55*10^5 s^2 + 1.104*10^7 s} \end{bmatrix}^T$$

With the ball held at the stationary point ($h_s = 0.01$m), the output response of the system for step input of $r = 0.01$ meter is shown in figure 8, as can be seen the closed-loop system with the proposed LQG controller performs well and the new stationary point is reached with no overshoot and with settling time ($T_s$) of about 0.592 sec, while the input voltage ($V$) have to be doubled to get the new stationary point (figure8-b). Also, the proposed controller achieves tracking excellently with little delay at the output of the system; this is evident in figure 9 which depicts the ball position of the system ($y$) for a sinusoidal input.

The designed controller present a good and fast disturbance rejection as shown in figure 10, where a step disturbance ($d$) of 0.02 at t= 15 sec is applied at the output of the system. In addition, the controller exhibits strong performance against system's parameter variations, the output of the system preserves its characteristics due to a variation of the circuit resistance ($R_C$) as shown in figure11.

Finally, figure 12 shows the output of the system for three different values of the weighting matrices $R$ and $Q$, as shown in the figure reducing $R$ yields faster response. On contrary, the output gets faster for larger values of $Q$.

$$Q_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 500 \end{bmatrix} \qquad ,$$

$$Q_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$
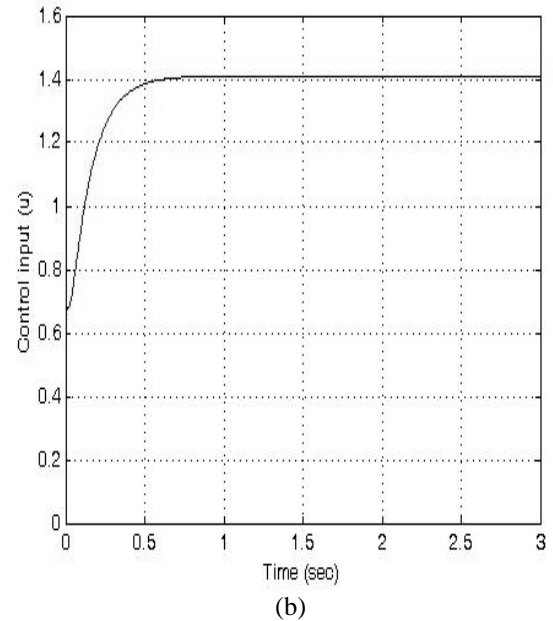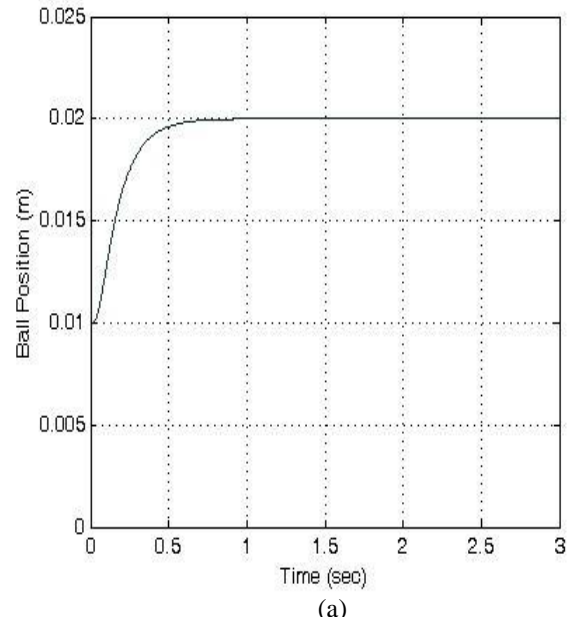


(a)



(b)

Figure 8: Output response of the magnetic levitation system, (a) Ball position, (b) Control input.
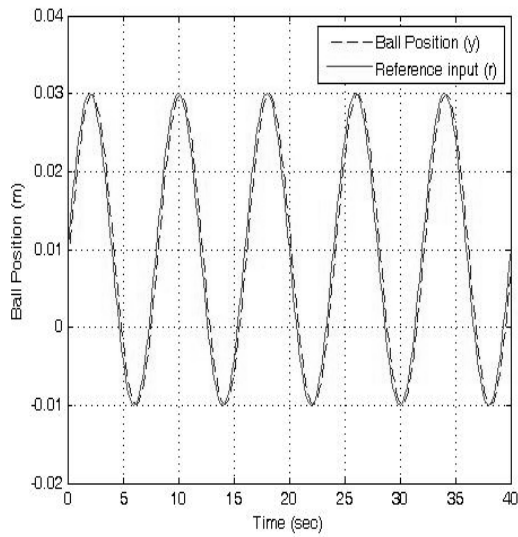
Figure 9: Output response of the magnetic levitation system due to sinusoidal input with amplitude of 0.02 meter and period of 8 sec.
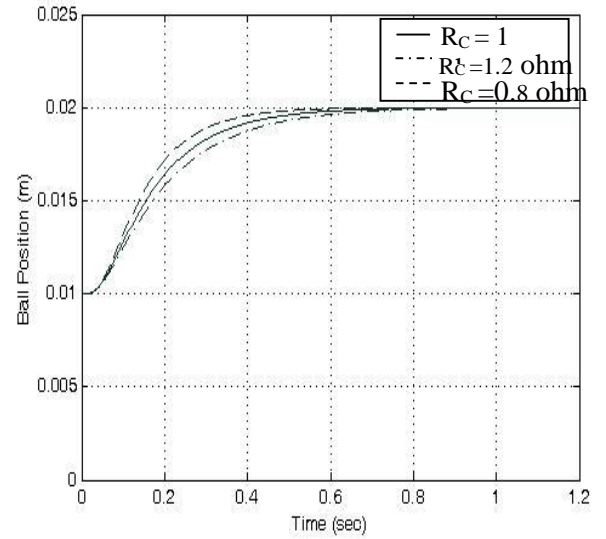


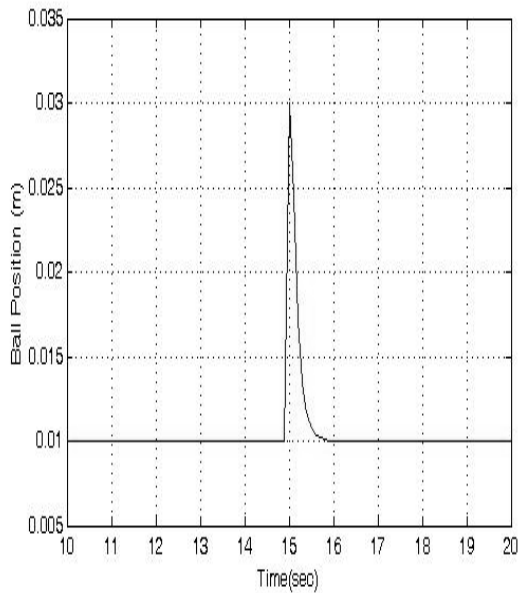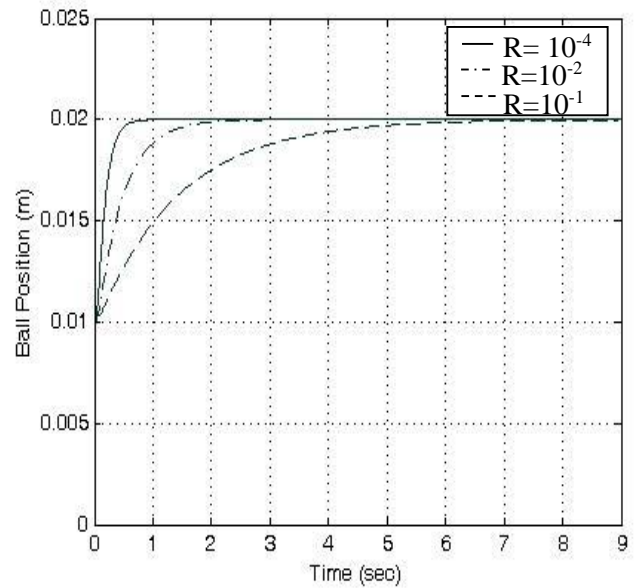Figure 11: Ball position for step change of 0.01 meter of the reference input (*r*) of the closed-loop system.



Figure 10: Ball position due to 0.02 meter step disturbance (*d*) at the output of the system.
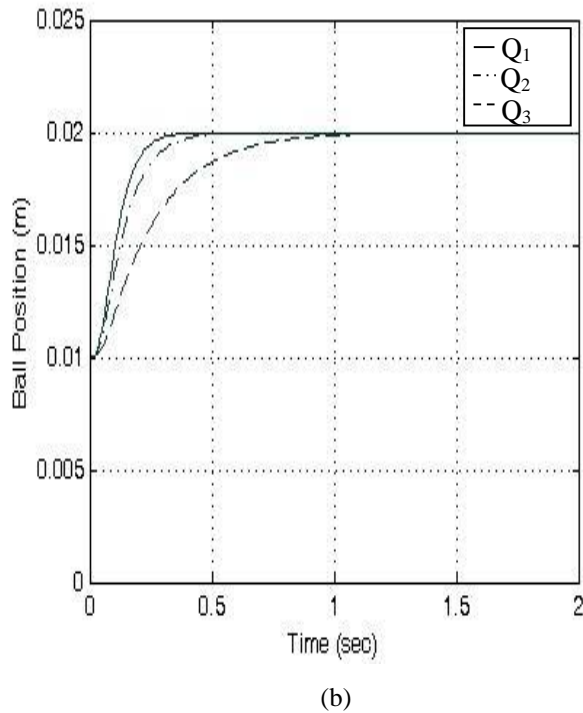


(a)

continued

(b)

Figure 12: Ball position for step change of 0.01 meter of the reference input (*r*) of the closed-loop system for different values of weighting matrices *R* and *Q*.

## VI.     Conclusions

this paperthe problem of a magnetic levitation control design is addressed and the following conclusions are made:

1. The analysis and design procedure of the proposed method using 2-DOF *LQG* control scheme is presented and a linearized state-space model of the magnetic levitation is derived.

2. It is found that the proposed controller perfectly track the reference input and attenuate the effect of the disturbance at the output of the system(see figure 8, 9, 10).

3. The proposed control scheme exhibits robust behavior to system parameter variations (see figure 11).

4. Finally, the design tuning parameters can be used to have a set of stabilizablecontrollers  and the one that presents the optimum  performance can be selected (see figure 12).

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