Natural Frequencies of Multi-Irregular Span Beams under Elastic Supports by Modal Analysis Method

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Abstract:

Evaluating the natural frequencies of multi-span beams with elastic supports play a major role in vibration designing and optimizing of many structures such as bridges, railways ,pipes and so on The continuity of the boundary conditions ,state space and numerical methods are normally used to investigate the vibration characteristics of such structures .Unfortunately ,such methods lead to high size matrix in dealing with the boundary value problem as the number of spans increase. In the present work, the problem is solved analytically by using Modal Analysis techniques in which the continuous system is discreteized to finite degree of freedoms in terms of the generalized coordinates A proper shape function are employed for describing the system dynamical behavior and satisfying the boundary conditions .In the present method the size of the resulting Eigen matrix depends on the number of mode chosen regardless of the number of spans. With this method wide variety of support configurations can be treated. The validly and convergence of the present method for calculating the natural frequencies is carefully checked by comparing with the exact values for two-span beams with different boundary conditions . It is found that using only (5) modes for the assumed solution gives only 2% error for two span simply supported and free ends beam , however for clamped ends the error is 8% .The present method is further checked by comparing with the Finite Element method the results show good agreements where the error is not increases 1% .The results of the natural frequencies of up to (10) equal and unequal spans beams under different boundary conditions and support stiffness are presented .The results showed that the natural frequencies can be highly controlled by proper choosing of the structure parameters and support stiffness.

Keywords: modal analysis ,muti-span beams ,natural frequency ,elastic support ,shape function

1-Introduction

The design of structural supports plays a key role in engineering dynamics and therefore close attention should be paid to their characteristics and number. Supports are not only expected to hold a structure firmly, but can also be redesigned to improve the structural performance. There are many engineering applications dealing with the dynamic of beams supported intermediately by rigid or elastic supports. Such multi span beams can be found in; bridges, rail ways, pipes, structure frames and so on.

The frequency equation of a beam with an intermediate support was developed by Rao [1], in which the continuity condition at the supported point was employed. The stiffness of an elastic single intermediate support for beams with different end conditions was investigated numerically by Wang [2].It is found that increasing the stiffness of the intermediate support leads to increase the natural frequencies . The frequency equation of a beamlike structure with regard to the position of a simple (or point) support was driven by using the discrete method. The effect of an intermediate support when the ends of the beam have elastic constraints was treated by Albarracı´n et al. [3]. It is concluded that the location of the support has significant effects on the natural frequencies especially for the odd number modes The effect of adding discrete masses on the beam natural frequencies for many end conditions was considered by Low [4].This study derived a closed-form solution for the minimum stiffness by using the derivatives of a natural frequency with respect to the support position. The solution process also provides insight into the dynamics of a beam with an intermediate support under general boundary conditions.

Timoshenko multi span beams carrying multiple spring-mass systems with axial force effect was analyzed by Yesilce [5].In this paper the problem was solved by using secant method for the non-trivial solution of different values of axial force .The effect of rotary inertia and shear deformation was investigated in this paper ,It is found that the effect of these parameters is to slightly increase the natural frequencies and it can be neglected for thin beams . The orthogonality conditions are used to solve the dynamical behavior of Euler -Bernoulli beam by Yozo [6]. In this paper a two-span beam with clamped–pinned–pinned supporting was investigated .The minimum stiffness of a simple support that raises a natural frequency of a beam to its upper limit for different boundary conditions was investigated by D. Wanga et al. [7].

Evaluating the eigenvalues of an arbitrarily supported single-span or multi-span beam carrying combination of lumped mass was performed by Philip and Cha [8] .The resulted frequency equation was formulated and solved numerically and graphically. Free vibration characteristics of a multi span beam with an arbitrary number of flexible constraints was investigated by Hai-Ping et al.[9]. Each span of the continuous beam was assumed to obey Timoshenko beam theory. The compatibility requirements on each constraint point were considered, the relationships between two adjacent spans was obtained. The Eigen solutions of the entire system then obtained by using a transfer matrix method.

The analysis of multiply continuously supported beams subjected to moving loads, which in turn can be modeled either as moving forces or moving masses was performed by DeSalvo et al. [10] . A dedicated variant of the component mode synthesis method was proposed .In this method the classical primary–secondary sub structure approach was tailored to deal with the slender (Euler–Bernoulli) continuous beams with arbitrary geometry. The whole structure is ideally decomposed in primary and secondary spans with convenient restraints. Numerical examples were presented to demonstrate the accuracy of the proposed procedure.

As it can be seen from the above review that there are sufficient methods for analyzing multi span beams with equal spans exists in the literature .However, there is a scarcity in the methods dealing with irregular spans beams (irregular supports spacing).In this paper a general procedures for evaluating the natural frequencies for any beam and supporting configurations will be presented .The procedure can be used for single and multi (equal or unequal) span beams.

2-Theoretical consideration

The model considered is a beam of length (L) and flexural rigidity (EI) seated on (N_s) number of elastic supports and obeyed Euler- Bernoulli theory .The ith elastic support is represented by translational and rotational springs with stuffiness constants of (K_{ti}) and (K_{ri}) respectively . As shown in Fig.(1).

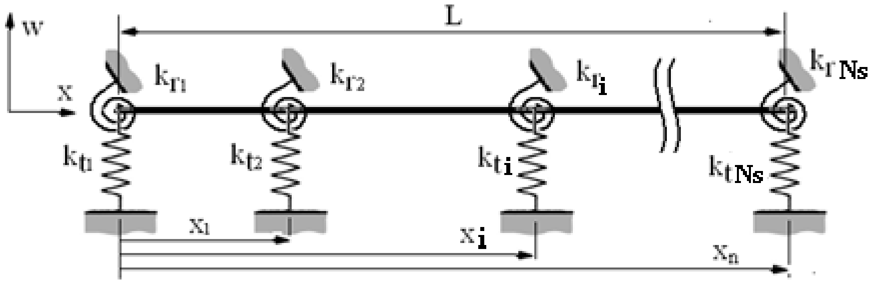


Figure 1: Beam seats on multi elastic supports

The analysis of the natural frequencies is started from writing the equations of motion of a beam subjected to external distributed loading , as the follows [11];

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial^2 t}=f(x,t) \quad \text{…. (1)}$$

Eq.(1) can be put into the following dimensionless forms;

$$\eta^{IV}+\ddot{\eta}=\alpha . f(\zeta,\tau) \quad \text{….. (2)}$$

Where;

$\eta = w/L$, $\alpha = L^3/EI$, $\zeta = x/L$ and

$$\tau = (t/L^2)\sqrt{EI/\rho A} \quad (3)$$

The notations $(.)'$ for $\partial/\partial\zeta$ and $(\dot{.})$ for $\partial/\partial\tau$ are used.

Eq.(2) may be discreatized by using Modal Analysis method. For this purpose a proper shape function for space must be chosen to satisfy all the boundary condition. In this work a modified shape function was tried to accomplish with the effects of the elastically deformed and undeformed modes. The undeformed modes are the rigid body translational and rotational modes.

Hence the suitable shape functions can be written as;

$$\eta(\zeta,\tau)=\sum_{s=3}^{N}\phi_s(\zeta)q_s(\tau)+\phi_T q_T+\phi_R q_R \quad ..... (4)$$

In the above equation, $\phi_s(\zeta)$ stand for the normal modes of beam free vibration (elastic), ϕ_T for rigid translational modes and ϕ_R for rigid rotational modes (non elastic) .

In the absence of all the constrained effects, the beam boundary conditions are free moment and shear force at both ends so that the normalized mode shapes $\phi_s(\zeta)$ can best chosen as free-free vibration normal modes .Such mode shapes take the following form [12];

$$\phi_s(\zeta)=\varphi_s(\zeta)=\sin\lambda_s\zeta+\sinh\lambda_s\zeta-\sigma_s(\cos\lambda_s\zeta+\cosh\lambda_s\zeta) \quad ..... (5)$$

Where;

$$\sigma_s=\frac{\sinh\lambda_s-\sin\lambda_s}{\cosh\lambda_s-\cos\lambda_s}, \quad ..... (6)$$

λ_s is the Eigen value of the free-free beam for s mode which are known ;

For example λ_1 =4.730041, λ_2 =7.853205, λ_3 =10.995608, λ_4 =14.1380,

The rigid translational and rotational modes can be normalized as;

$$\phi_T=1,\phi_R=\zeta \quad ..... (7)$$

Substituting, Eq. (7) into (4) leads to ;

$$\eta(\zeta,\tau)=\sum_{s=0}^{N}\phi_s(\zeta)q_s(\tau)+q_T+\zeta.q_R \quad .... (8)$$

Now, substituting Eq. (8) into Eq. (2) gives;

$$\sum\phi_s^{IV}q_s+\sum(\phi_s\ddot{q}_s+\ddot{q}_T+\zeta.\ddot{q}_R)=\alpha.f(\zeta,\tau) \quad ..... (9)$$

Where $q(\tau)$ is replaced by q for simplicity

Now, multiplying Eq. (9) by the boundary residual series $\phi_r(\zeta)=\sum_{r=1}^{N}\phi_r(\zeta)+1+\zeta$ and integrating over the whole beam length (0 to 1) , the following matrix equation can be obtained ;

$$[A]\{q\}+[M]\{\ddot{q}\}=\alpha\{F\} \quad ..... (10)$$

Due to the orthogonally property of the normal modes which are;

$$\int_0^1\phi_s(\zeta)\phi_r(\zeta)d\zeta=\begin{cases}1\,for...s=r\\0\,for....s\neq r\end{cases} \quad ..... (11)$$

and;

$$\int_0^1\phi_r^{IV}(\zeta)\phi_s(\zeta)d\zeta=\begin{cases}\lambda_s^4\,for...s=r\\0\,for...s\neq r\end{cases}$$

$$\int_0^1\zeta\zeta.d\zeta=1/3 \quad ..... (12)$$

The elements of matrices [*A*] and [*M*] are given in the Appendix B .

In equation (2) the forcing term $f(\zeta,\tau)$ represents all the generalized elastic forces excreted on the beam due to the effects of the linear and torsional springs .

Referring to Fig. (1), the i[th] linear and torsional spring force and moment on the beam can be written as;

$$f_i=-k_{Ti}y(\zeta_i,\tau)$$

$$=-k_{Ti}\sum_r^N q_r\phi_r(\zeta_i)+q_T+q_R\zeta_i \quad ..... (13)$$

$$\mu_i=-k_{Ri}y'(\zeta_i,\tau)=-k_{Ri}\sum_r^N q_r\phi_r'(\zeta_i)+q_R \quad ..... (14)$$

From these equations the generalized forces {F}vector in Eq.(10) can be found as the follows;

1-For linear spring;

$$F_T^i=\int_0^1-k_{ti}(\sum_r^N q_r\phi_r(\zeta_i)+q_T+q_R\zeta_i)\{\sum_s^N\phi_s(\zeta)+1+\zeta\}\delta(\zeta-\zeta_i)d\zeta$$

$$=-k_{ti}(\sum_r^N q_r\phi_r(\zeta_i)+q_T+q_R\zeta_i)\{\sum_s^N\phi_s(\zeta_i)+1+\zeta_i\} \quad ..... (15)$$

2-For torsional spring;

$$F_R^i=\int_0^1-k_{ri}(\sum_r^N q_r\phi_r'(\zeta_i)+q_R)\{\sum_s^N\phi_s'(\zeta)+1\}\delta(\zeta-\zeta_i)d\zeta \quad ..... (16)$$

$$=-k_{ri}(\sum_r^N q_r\phi_r'(\zeta_i)+q_R)\{\sum_s^N\phi_s'(\zeta_i)+1\} \quad ..... (17)$$

The Diarac delta δ is used since the forces are concentrated. Substituting Eqs.(15) and (16) into Eq.(10) and arranging gives;

$$[A]\{q\}+\alpha\sum_i^{Ns}(k_{ti}[H^i]+k_{ri}[L^i])\{q\}$$
$$+[B]\{\ddot{q}\}=0 \quad ..... (18)$$

Where, *Ns* denotes the number of supports or constrained points of the beam.
The elements of matrix $[H^i]$ and $[L^i]$, are given in Appendix C

Finally, Eq. (18) can be put in the following standard form;

$$[K]\{q\}+[M]\{\ddot{q}\}=0 \quad \text{….. (19)}$$

Where;

$$[K]=[A]+\sum_{i}^{Ns}(K_{ti}[H^i]+K_{ri}[L^i]) \quad \text{…..(20)}$$

Where ;$K_{ti} = \alpha k_{ti}$ and $K_{ti} = \alpha k_{ti}$

Are the dimensionless translational and rotational stiffness.

Eq.(20) tells that increasing the number of support has no effect on the size of the matrix [K] since it depend on the number of modes choosen .In many of the other methods like the continuity of boundary conditions and Finit Element the corresponding matrix size increased with the number of supports . This is the main advantage of the present method which allows for analyzing any number of span beams in one program .

Since vibration is harmonic motion one can assume the following solutions for *q*;

$$\{q\}=\{\hat{q}\}e^{i\omega\tau} \quad \text{….. (21)}$$

Where $\{\hat{q}\}$, is an arbitrary vector.

Substituting Eqs. (21) Into Eqs. (19) and eliminating the arbitrary constants yield to the following determinant ;

$$\left|[K]-\omega^2[M]\right| = 0 \quad \text{….. (22)}$$

Equation (22) can be used to investigate the natural frequencies for the following cases ;

1. Single span beam under classical boundary condition.
2. Single span beam under elastic supports.
3. Multi span beam with equal or unequal span under different boundary conditions.
4. Beam under multi elastic supports.

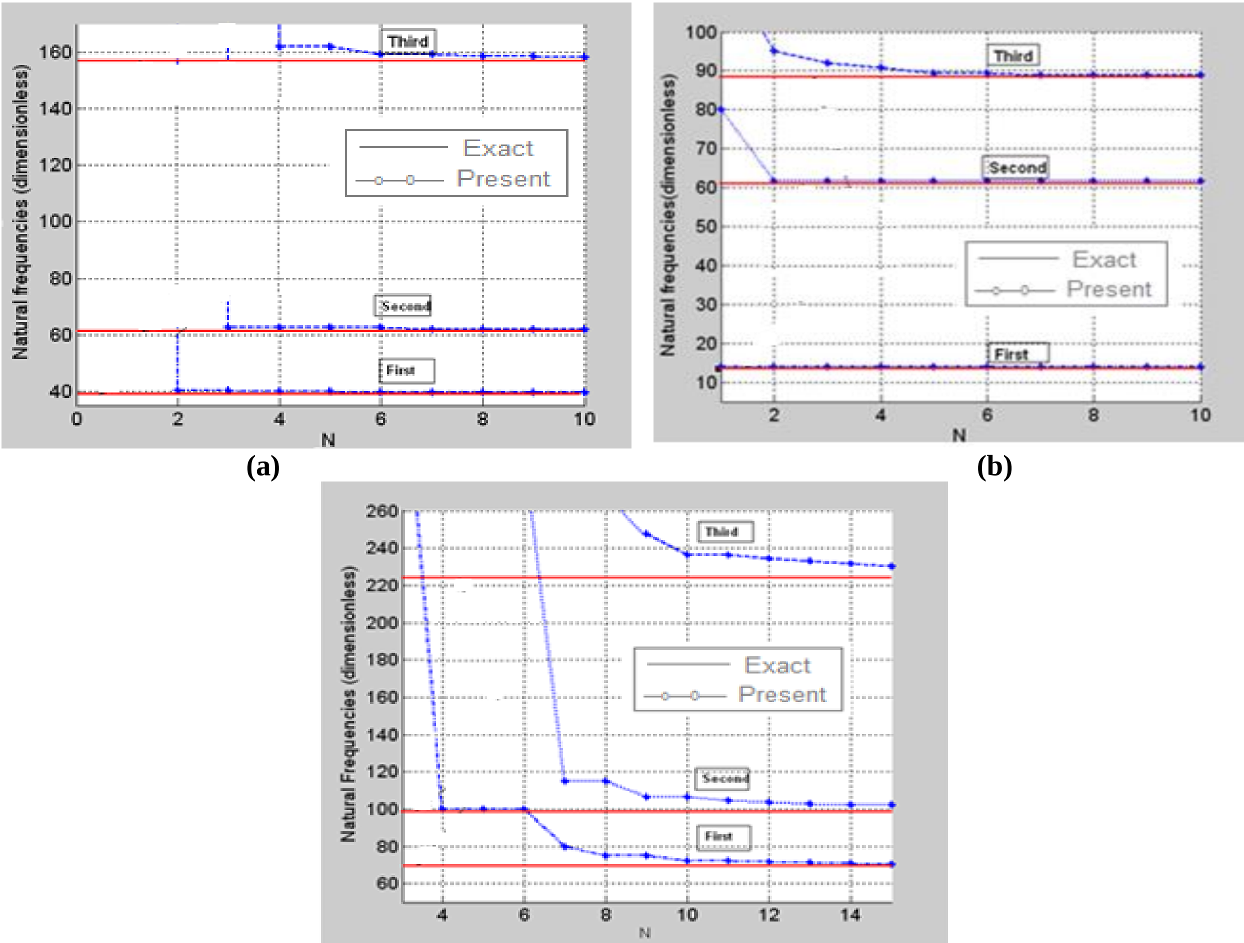
By the successive choice of the parameters (Ns,ζ K_{ti} and K_{ri}) the upper four cases can be obtained .For example a three-equally spans beam under f-s-s-c supporting one must set (Ns=3; ζ =0,⅓,⅔,1 ; K_{ti} = 0,1x10^{12},1x10^{12},1x10^{12};and K_{ri}=0,0,1x10^{12}),in which the value 1x10^{12} is assigned for rigid supports (numerical infinite stiffness) .

3-Results and discussions

Prior of any calculation, the validity of the assumed shape functions and the convergence of the present method were tested. In this test , the “exact” values of the natural frequencies for the lowest three modes of two-span beams under s-s , f-f and c-c boundary conditions were calculated by using the method of continuity of the boundary conditions(see for example ref.11). The resulting Eigen matrices for the three cases are given in Appendix D .The convergence and the accuracy of the present method are tested with aid of Figs.(2) .In these figures the values of the lowest three natural frequencies of the above mentioned beams are evaluated by the present method with the number of the assumed modes (N) are varied from (1 to 10) for s-s and f-f beams and from (1-15) for c-c beam .The exact values of the natural frequencies are given in the same figure ,also .The figures show that in general the accuracy of the solution is improved as the number of the assumed modes increased .In case of s-s and f-f beams (Figs.2-a,b) the natural frequencies are rapidly converge toward the exact values for the three considered modes. and when (N=6) an accuracy of 98% can be obtained .However for c-c beam the required number of modes to achieve satisfactory accuracy is higher .For example to achieve 92% accuracy one must use (N=10) for the first mode ,(N=13) for the second mode and (N =15) for third mode . The reason of this behavior can be attributed to the nature of the assumed shape functions .This solution generate elastic curves which are more closer to the exact curve for s-s and f-f beams than that of c-c beam .

To check the validity of the present solution, a beam with (1 to 10) spans simply supported at both ends with rigid intermediate supports is solved by Finite Element Method (FEM) and by the present method, also .The ANSYS 14 software is employed for solving the FEM in which the beam spans are represented by using BEAM3 element while the translational and rotational springs are represented by using COMBIN14 element. The result of the two methods are collected in Table (1).As it can be seen form the table that the results are in a very good agreements where the maximum error is not exceeded 0.6% for the worst case .

The effect of increasing the number of spans on the lowest four natural frequencies are shown in Figs.(3-a,b,c) .In these figures beams under s-s, c-f and c-c boundary conditions with intermediate simply supported are investigated .As it can be seen that ,the natural frequencies are increased as the number of span increased for



(a) (b)

(c)

Figure 2: Convergence test for two-span beams under (a) s-s ,(b) f-f and (c) c-c boundary conditions

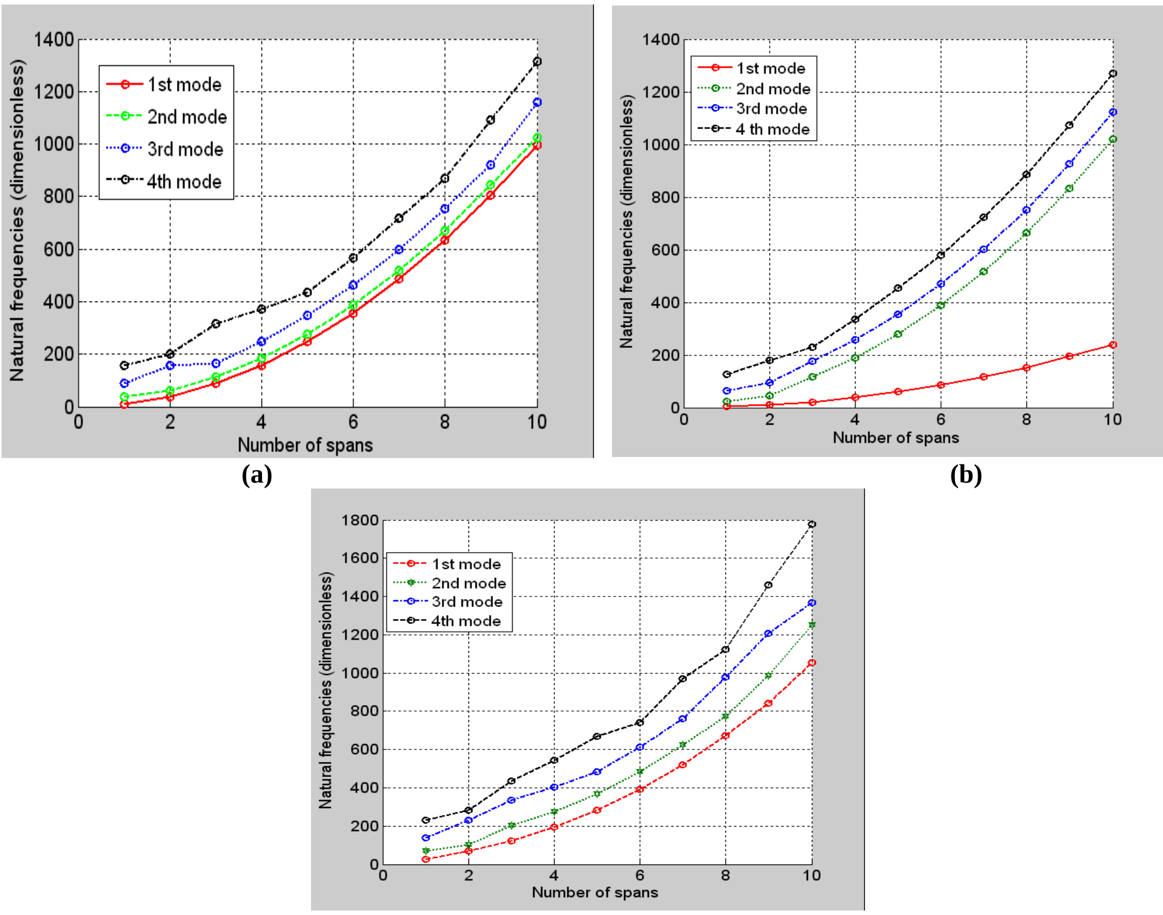
All boundary conditions .This can be reasoned due to the fact that as the number of supports increase the spans become shorter and stiffer. .The higher stiffness leads to higher natural frequency .The irregularity in curves for the higher mode may be due to the error in the accuracy which increase in such modes as stated earlier.

The effect of the stiffness of the intermediate supports on the lowest four natural frequencies of (1-10) span spans beam are investigated in Figs(4-a,b,c).From these figures it is clear that increasing support stiffness lead to increase the natural frequencies for all modes .

Moreover one can see that for higher stiffness (K_t=1000) the natural frequencies increase more clearly. This indicates that, using rigid supports can effectively raise the natural frequencies to higher values

Table 1: Compression of natural frequencies of (1 to 10) span beam simply supported at both ends with ANSYS

N_{sp}	1st mode			2nd mode			3rd mode			4th mode		
	FEM	present	E%	FEM	present	E%	FEM	present	E%	FEM	present	E%
1	9.7331	9.7342	0.0113	39.5010	39.516	0.0380	89.3960	89.499	0.1152	157.5690	158.21	0.4068
2	39.515	39.5160	0.0025	61.7041	61.72	0.0258	158.1025	158.21	0.0680	200.9851	202.6	0.8035
3	89.48	89.495	0.0168	114.1215	114.16	0.0337	166.5040	166.72	0.1297	354.2571	356.95	0.7602
4	158.201	158.2234	0.0142	186.6637	186.69	0.0141	248.0333	248.28	0.0995	317.1452	320.73	1.1303
5	247.841	247.8430	0.0008	275.6235	275.8	0.0640	346.2108	346.69	0.1384	430.2653	434.41	0.9633
6	356.911	356.9251	0.0040	386.7175	386.95	0.0601	461.4215	461.89	0.1015	562.6875	567.75	0.8997
7	486.0321	486.0588	0.0055	518.4273	518.77	0.0661	596.3025	596.96	0.1103	710.1026	715.96	0.8249
8	635.8301	635.8442	0.0022	667.8516	668.24	0.0582	754.0123	754.73	0.0952	861.2003	867.06	0.6804
9	803.523	803.6552	0.0165	843.2464	843.64	0.0467	920.1541	920.98	0.0898	1087.2311	1093	0.5306
10	994.7103	994.8507	0.0141	1025.137	1025.7	0.0549	1159.1140	.1160	0.0764	1305.7834	1313.9	0.6216



(c)

Figure 3: Variation of natural frequencies of the lowest modes for (a)s-s ,(b)c- f (c) c-c beams with the number of equal spans

This is one of practical solutions to avoid the dangerous effect of the resonance by increasing the natural frequency .The natural frequencies for multi unequal spans beams under s-s ends are shown in Figs(6-a,b) .In Fig.(6-a) the dimensionless span lengths are assigned sequence values of (0.018, 0.036, 0.052,…..,0.18).In Figs (6-b) they assigned;(0.07, 0.14, 0.07, 0.14 , …. ,0.14).An example to illustrate the later sequence is shown schematically in Fig.(5).As it can be seen from comparing the two figures that the relation between the number of spans and the natural frequencies can be quite different .This means that an optimization can be made by proper choosing of the intermediate supports locations .

For the two studied cases one can see that the case of Fig(6-b) is more effective in increasing the natural frequencies than that of Fig.(6-a) .For example one needs to construct (6) spans to increase the first dimensionless natural frequency to (150) by using the first configuration while needs (8) spans to achieve the same frequency for the second .

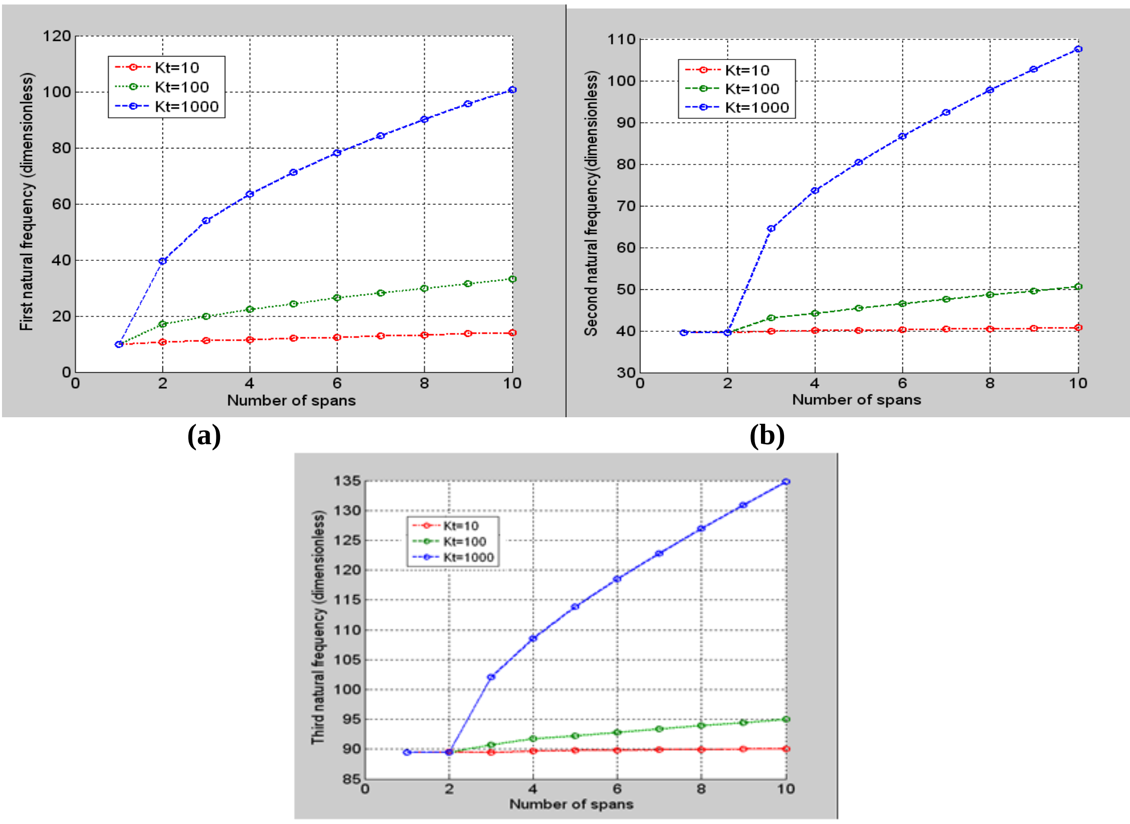


Figure 4: Variation of natural frequencies with the number of spans of s-s beam for (a)first ,(b)second and (c)third modes, at different translational stiffness

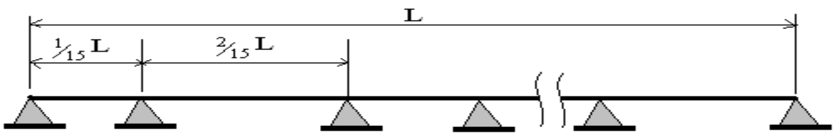
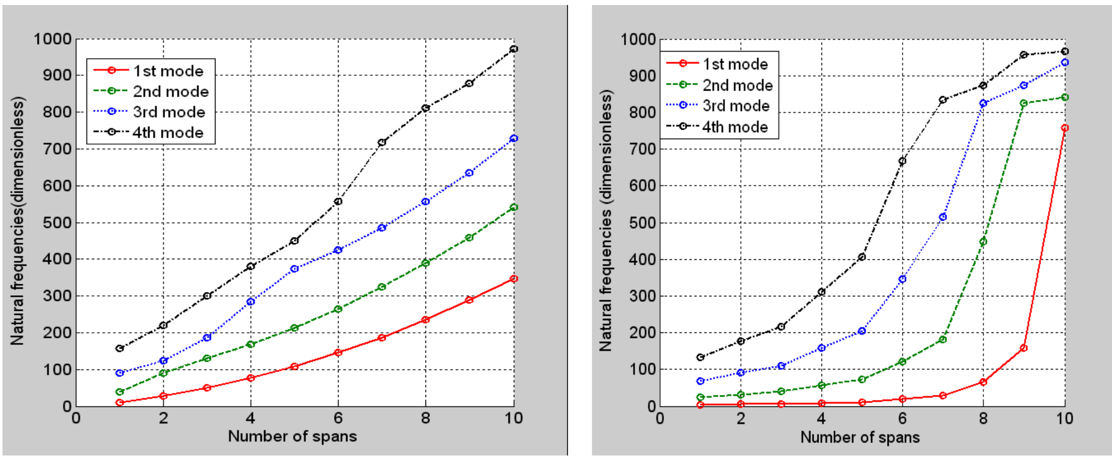


Figure 5: Unequal span beam configuration with span length in sequence of (0.07,0.14,0.07,…0.14)



(a) **(b)**

Figure 6: Variation of natural frequencies of the lowest modes for unequal span beams have sequence; (a) 0.018,0.036,0.054,…0.18 and (b) 0.07,0.14,0.07,…0.14

A general case of multi span beam, in which the span length are not equal and all the supports has translational and rotational stiffness ,is investigated in Fig.(7). In this figure the lowest four natural frequencies are plotted with dimensionless span lengths of; (0.07, 0.14, 0.07……0.14) .The figure shows that the natural frequencies increasing depend on the support stiffness and the spacing. This situation can aid the designer to control the natural frequencies by considering these parameters.

As it is clear from the above considered cases that wide variety of beam configurations (span length and number, boundary conditions, supports elasticity) can be simply and effectively investigated by using the presented procedures.

The present procedure can offer a comprehensive investigation for the design and optimization requirements for multi-span beams (multi supported) dynamic by adjusting the natural frequencies .

4-Conclusions

From the discussion of the results and the comparison with the exact and the FE methods, the following conclusions can be summarized as the follow;

1- The present method shows a good convergence and accuracy as compared with the exact method .It is found that; acquiring a good accuracy required only few number of assumed modes depending on the boundary conditions. For s-s and f-f end supporting five modes are sufficient, however, for clamped ends (10-15) modes must be considered.
2- .Many classes of problems related with multi- span beams can be treated with the present method easily and effectively.
3- The size of the resulting Eigenvalue matrix depends on the number of mode chosen regardless of the number of spans as it is the case for the other corresponding methods .
4- It is found that increasing support stiffness and span number lead to increase the natural frequencies for all modes under different boundary conditions.
5- The natural frequencies can be successively controlled by proper adjusting the support and beam parameters. Hence, the present method can offer simple and unique procedures for designing and optimizing requirements for multi- span beam structures .

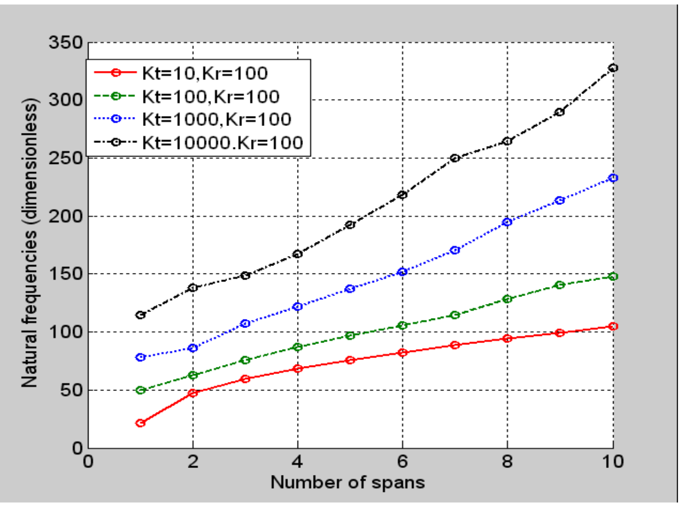


Figure 7: Variation of natural frequencies of the lowest modes with different elastic supports for unequal spans beam s-s ends

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Appendix

A) Notations;

s-s :simply supported ends beam

f-f :free ends beam

c-c :clamped ends beam

c-f :clamped-free ends beam

B) elements of matrices [A] and [M] ;

$$[A]=\begin{bmatrix} \lambda_1^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2^4 & 0 & 0 & 0 & 0 \\ 0 & 0 & .... & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_N^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$[B]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & .... & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 \end{bmatrix}$$

C) Elements of matrix $[H^i]$ and $[L^i]$,;

$$h^i{}_{r,s} = \phi_r(\zeta_i)\phi_s(\zeta_i)$$

$$h^i{}_{1+N,s} = \phi_s(\zeta_i)$$

$$h^i{}_{1+N,1+N} = 0,$$

$$h^i{}_{2+N,2+N} = 1$$

$$l^i{}_{r,s} = \phi_r(\zeta_i)\phi_s(\zeta_i),$$

$$l^i{}_{1+N,s} = 0,$$

$$l^i{}_{1+N,s} = \phi_s(\zeta_i)$$

$$l^i{}_{1+N,1+N} = 0, l^i{}_{2+N,2+N} = 1.$$

D) The Eigen matrix of two-span by using the “exact” solution

(i) For f-f ends supports;

[-1 0 1 1 0 0 0 0
0 -1 0 1 0 0 0 0
sin (λs) cos(λs) sinh(λs) cosh(λs) 0 0 0 0
cos(λs) -sin(λs) cosh(λs) sinh(λs) -1 0 -1 0
-sin (V) -cos(λs) sinh(λs) cosh(λs) 0 1 0 -1
0 0 0 0 0 1 0 1
0 0 0 0 cos(λs) sin(λs) cosh(λs) sinh(λs)
0 0 0 0 -sin(λs) -cos(λs) sinh(λs) cosh(λs)]

(ii) for s-s end supports ;

[0 1 0 1 0 0 0 0
1 0 1 0 0 0 0 0
sin (λs) cos(λs) sinh(λs) cosh(λs) 0 0 0 0
cos(λs) - sin(λs) cosh(λs) sinh(λs) -1 0 -1 0
-sin (λs) -cos(λs) sinh(λs) cosh(λs) 0 1 0 -1
0 0 0 0 0 1 0 1
0 0 0 0 sin (λs) cos(λs) sinh(λs) cosh(λs)
0 0 0 0 -sin (λs) - cos(λs) sinh(λs) cosh(λs)]

(iii) For c-c end supports;

[0 1 0 1 0 0 0 0
0 -1 0 1 0 0 0 0
Sin (λs) cos(λs) sinh(λs) cosh(λs) 0 0 0 0
cos(λs) -sin(λs) cosh(λs) sinh(λs) -1 0 -1 0
-Sin (λs) -cos(λs) sinh(λs) cosh(λs) 0 1 0 -1
0 0 0 0 0 1 0 1
0 0 0 0 sin (λs) cos(λs) sinh(λs) cosh(λs)
0 0 0 0 -sin (λs) -cos(λs) sinh(λs) cosh(λs)] ,

Where λs are the Eigen values

الترددات الطبيعية للعتبات الغير منتظمة الفضوة المتعدده تحت اسناد مرن باستخدام التحليل الشكلي

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الخلاصة:

ان ايجاد الترددات الطبيعية للعتبات ذات الفضاءات المتعدده والموضوعة على مساند مرنة يلعب دورا اساسيا في التصميم الامثل للاهتزاز للعديد من الهياكل كالجسور وخطوط السكك الحديدية والانابيب وغيرها .تستخدم عدة طرائق لبحث خصائص الاهتزاز مثل استمرارية الظروف المحيطية وحيز الحالة والطرائق العددية . تنتج عن تلك الطرائق مصفوفات كبيرة الحجم تزداد بزيادة عدد الفضاءات .في العمل الحالي تم حل المشكلة تحليليا باستخدام تقنيات التحليل الشكلي Modal Analysis حيث تم فيها تجزئة النظام المستمر الى عدد محدد من درجات الحرية بدلالة الاحداثيات المتولدة . استخدمت دوال شكلية ملائمة لوصف التصرفات الديناميكية للنظام واستيفاء الظروف الحدية .في هذه الطريقة يعتمد حجم المصفوفة الذاتية الناتجة على عدد الانساق المختارة بغض النظر عن عدد فضاءات العتبة وبذلك يمكن التعامل مع انواع مختلفه من المساند.تم تدقيق سريان مفعول هذه الطريقه في حساب الترددات الطبيعية وذلك بالمقارنة مع القيم الحقيقية وذلك لنموذج عتبة ذات فضائين تحت ظروف اسناد مختلفة فظهر ان افتراض خمسة انساق للحل يسبب نسبة خطأ بحدود 2% في حالة الاسناد البسيط و8% للاسناد المحكم.كما تم تدقيق الطرية بالمقارنة مع طريقة العناصر المحددة FEM فبينت النتائج توافقا جيدا حيث لم تتعدى نسبة الخطأ 1%.تم استعراض نتائج لعتبات مكونة من عدد يصل لغاية 10 فضاءات ذات اطوال متساوية أومختلفة وتحت مساند مرنة فبينت النتائج انه بالمكان السيطرة على الترددات الطبيعية بشكل كبير اعتمادا على خصائص النظام ومرونة المساند .

الكلمات المفتاحية : التحليل الشكلي , عتبة متعددة الفضاءات ,التردد الطبيعي ,المساند المرنة ,دالة الشكل.