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Sliding Mode Controller for Electromechanical System with Chattering Attenuation

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Abstract:

Electromechanical systems (EMS) may be considered as devices transforming electrical into mechanical energy. Every system that belongs to the electromechanical class can be decomposed in an electrical (ES) and a mechanical subsystem (MS). The motion control systems can be quite complicated because many different factors have to be considered in the design of electromechanical systems. These factors can be summarized as the nonlinearity, non-smoothness in its model, the uncertainty in system model parameters and non-satisfying matching condition.

In this paper a new sliding mode control design approach for the EMS is proposed without neglecting the inductance in the electrical part or approximating the non-smooth perturbation. The first step in the proposed controller design consists of transforming the ES to a low pass filter (LPF) and then (the second step) designing a sliding mode controller (SMC) to the MS that will reject system model uncertainty and the effect of non-smooth disturbances. With a suitable selected LPF time constant, the SMC which controls the MS is nearly the equivalent control and as a result the chattering is attenuated greater than that in the case of classical SMC which designed by ignoring the electrical subsystem and also with a smaller control effort. The simulation results, of applying the proposed sliding mode control to an electromechanical system, show its superiority compared with classical SMC designed in two effective SMC features beside forcing the state to follow the desired position where chattering amplitude is greatly reduced with a significant reduction in control action value (approximately equal to third the required input voltage with the classical SMC).

Keywords: Electromechanical systems, Sliding mode control, Low pass filter, Chattering attenuation

1. Introduction:

Electromechanical systems may be considered as devices transforming electrical into mechanical energy. They establish a very important class of industrial components often found in current practical applications defining the ideal link between computer-based drivers and movement generators systems. From the control point of view their structure is quite interesting since they belong to the class of the so-called under actuated systems in the sense that only the electrical part is directly actuated; the mathematical models that represent their dynamical behavior are, in general, nonlinear and, in many cases, the state is not completely available for measurement [1]. The main assumption is that every system that belongs to the electromechanical class can be decomposed into an electrical and a mechanical subsystem which it can be viewed as the ***interconnection*** of electrical and mechanical lumped elements, respectively. For the ***electrical subsystems*** these elements are inductances, capacitances and resistances, while for the ***mechanical subsystem*** are springs, masses and dampers [1].

Motion control is concerned with manipulating power to control the movement of a mechanical system. A large amount of motion control is now performed using electric motors, so that it will be our main focus. Motion control systems can be quite complicated because many different factors have to be considered in the design. The following issues must typically be considered:

- Reduction of the influence of plant disturbances
- Attenuation of the effect of measurement noise
- Variations and uncertainties in plant behavior

It is difficult to find design methods that consider all these factors, especially for the conventional control approaches where control designs involve compromises between conflicting goals. In order to design control systems to get high performance and robustness when controlling such complicated processes, advanced controllers have been introduced.

From control design point of view there are mainly three problems for the application of many control theories which can be summarized as follows;

1) The nonlinearity and non-smoothness in its model due to the friction model and the nonlinear spring.

2) The uncertainty in system model parameters.
3) The matching condition is not satisfied (where the perturbations that enter the state equation do not at the same point as the control input).

The solutions of these problems, which exist in literature, can be classified to two types:

In the first, the electrical part is ignored (the inductance in the mathematical model of the DC motor is ignored), consequently this leads to delay in the performance of the system [2], [3], [4], [5].

The second solution is focused when the electrical part is not neglected and the external and un-modeled dynamics (the perturbation) are smooth. For the smooth perturbation the control design uses a systematic Backstopping [6], but when the perturbation is not smooth in this situation the best that can be done is to regulate the state to a positively invariant set including the origin and stay there for all future time [7].

Another solution for control design is by using Backstopping but after approximating the perturbation (making it smooth) [8]. On the other hand, the complete electromechanical system model can be used for control design but with nominization of the system model to a nonlinear canonical form [9].

The sliding mode control method, can be solved the first and second problems since it can deal with these types of nonlinearity and non-smoothness in affecting system behavior. The third problem is frequently arises in the electromechanical system where the uncertainty in system model and the disturbances does not appear in the control channel. The effect of the mismatched uncertainty and disturbances can be attenuated, but not eliminated, as in case of using integral sliding mode control (ISMC) [10].

In this paper a new approach is suggested for an electromechanical system control design without neglecting the electrical subsystem (ES) or approximating the non-smooth perturbation. First the controller is designed to transform the ES to a low pass filter (LPF) with a suitable small time constant. Then a control law is derived for the mechanical system using sliding mode control theory which it is able to force the mechanical state to the desired reference in spite of the presence of uncertainty and external disturbances in the electromechanical system model. The simulation results for a typical electromechanical system in the subsequent sections will demonstrate the effectiveness of the proposed approach.

2. Electromechanical Model Description:-

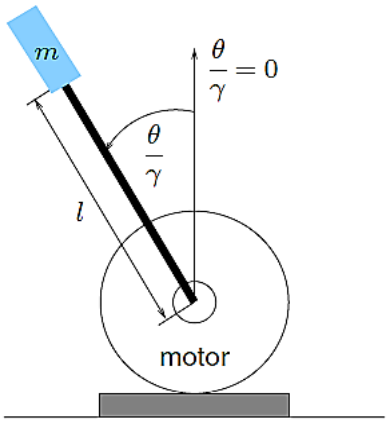

Figure 1: Schematic diagram of the considered electromechanical sample system, an inverted pendulum actuated by a DC motor [11].

The DC-motor can be mathematically modeled by using the dynamic equivalent circuit of DC-motors. The voltage equation of the armature circuit under transient given by the following equation [11]:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + v_b \quad \ldots\ldots (1)$$

where:
v_a-the source voltage (voltage)
i_a-the motor armature current (Amber)
$v_b = k_n w_m$-the back emf (voltage)

From the dynamics of motor load system:

$$J\frac{d\omega_m}{dt} = T - T_L - c_1\omega_m \quad \ldots\ldots (2)$$

where [11]

$$\left.\begin{aligned} T &= k_m i_a \\ T_L &= -\frac{1}{\gamma} mgl \sin(\frac{\theta}{\gamma}) \end{aligned}\right\} \quad \ldots\ldots (3)$$

where:
T,T_L- are the motor electromagnetic torque and the mechanical torque of the load, respectively and θ-the angle of the pendulum with the upright position being zero. The other parameters are described in Table (4.1)

Now let $x_1 = \theta$, $x_2 = \dot{\theta} = \omega_m$, $x_3 = i_a$ and the manipulated variable $u = v_a$ then, the mathematical model in Eqns. (1), (2) can be rewritten as follows:

$$\dot{x}_1 = x_2 \quad \ldots\ldots (4)$$

$$\dot{x}_2 = \frac{1}{J}(k_m x_3 - c_1 x_2 + \frac{1}{\gamma} mgl \sin(\frac{x_1}{\gamma})) \quad \ldots\ldots (5)$$

$$\dot{x}_3 = \frac{1}{L}(u - R_a x_3 - k_n x_{2)} \quad \ldots\ldots (6)$$

Let: $a_{21} = \frac{1}{J\gamma} mgl$, $a_{22} = \frac{c_1}{J}$, $a_{23} = \frac{k_m}{J}$, $a_{32} = \frac{k_n}{L}$, $a_{33} = \frac{R_a}{L}$, and $b_3 = \frac{1}{L}$ then the system becomes:

$$\dot{x}_1 = x_2 \quad \text{..... (7)}$$
$$\dot{x}_2 = a_{21} \sin\left(\frac{x_1}{\gamma}\right) - a_{22}x_2 + a_{23}x_3 \quad \text{..... (8)}$$
$$\dot{x}_3 = -a_{32}x_2 - a_{33}x_3 + b_3 u \quad \text{..... (9)}$$

Where Eqs (7) and (8) are for the mechanical subsystem (MS) and Eq. (9) is for the electrical subsystem (ES).

3. Sliding Mode Control:

Sliding-mode control (SMC) is a robust technique, well known for its ability to reject the external disturbances and model uncertainties satisfying the *matching condition*, that is, perturbations that enter the state equation at the same point as the control input. SMC has other advantages as well, like ease of implementation and reduction in the order of the state equation. The conventional SMC design methodology comprises two steps:

First, design a sliding manifold (named also as switching manifold) such that the system's motion along the manifold meets the specified performance. **Second** design a (discontinuous) control law such that the system's state is driven toward the manifold and stays there for all future time, regardless of disturbances or uncertainties

In spite of the robustness of SMC against the matched disturbances and ease of implementation, but it has **two main disadvantages**:

The first is in the case of mismatched disturbances. F. Castaños and L. Fridman suggest the use of the integral sliding mode to reject the matched disturbances and the $H\infty$ techniques to attenuate the unmatched one [12]. The Integral Sliding Mode Control (ISMC) is also applied for the nonlinear Systems with matched and mismatched perturbations by Matteo Rubagotti et.al. [10]. **The second** problem is the chattering behavior which frequently appears in sliding mode control system for many reasons such as the non-ideality of the switching process as shown in the excellent reference by **V. I. Utkin** [2009][13].

In the following subsection is given the design of sliding mode controller for the electromechanical system that includes a primary design step devoted to transform the electrical subsystem to a low pass filter with the desired time constant.

3.1 SMC Design for electromechanical system:

In this section a new SMC is designed to the electromechanical system that taken into consideration the uncertainty in system model and the presence of the nonlinearity. Moreover the mismatch and chattering problems are considered where the controller designed first to transform the electrical subsystem to a low pass filter with desired time constant. Now the mathematical model of the electromechanical system as derived in section two is given by

$$\dot{x}_1 = x_2 \quad \text{..... (10)}$$
$$\dot{x}_2 = f_2(x) + g_2(x)x_3 \quad \text{..... (11)}$$
$$\dot{x}_3 = f_3(x) + g_3(x)u \quad \text{..... (12)}$$

where $f_2(x) = a_{21} \sin\left(\frac{x_1}{\gamma}\right) - a_{22}x_2$, $g_2(x) = a_{23}$, $f_3(x) = -a_{32}x_2 - a_{33}x_3$, and $g_3(x) = b_3$.

The **first step** in the present design for the sliding mode control is to transform the electrical subsystem (Eq. (12)), to a low pass filter as follows;

Let, $f_3(x) + g_3(x)u = \frac{1}{T}(-x_3 + \vartheta)$ ..... (13)

Then Eq. (12) becomes:

$$\dot{x}_3 = \frac{1}{T}(-x_3 + \vartheta) \quad \text{..... (14)}$$

Where T is the time constant (selected) for the Low Pass Filter (LPF) induced by the controller u where it is assumed that f_3, g_3 are known without uncertainty. Accordingly the system dynamics becomes:-

$$(MS)\begin{cases}\dot{x}_1 = x_2 & \text{..... (15)}\\ \dot{x}_2 = f_2(x) + g_2(x)x_3 & \text{..... (16)}\end{cases}$$
$$(ES)\left\{\dot{x}_3 = \frac{1}{T}(-x_3 + \vartheta)\right. \quad \text{..... (17)}$$

To this end replace x_3 by ϑ in Eq. (16) the sliding mode controller for the MS may then be designed as follows; dividing the system into nominal part and uncertainty part

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f_{2o}(x) + g_{2o}(x)\vartheta + \delta(x,u)\end{aligned}\right\} \quad \text{..... (18)}$$

Where $f_{2o}(x)$ and $g_{2o}(x)$ are the nominal functions of $f_2(x)$ and $g_2(x)$ respectively while $\delta(x,u)$ is the uncertainty term resulting from the uncertainty in system dynamics. $\delta(x,u)$, which is given by

$$\delta(x,u) = \Delta f_2(x) + \Delta g_2(x)\vartheta \quad \text{..... (19)}$$

Let the sliding variable s (named also in the literature as switching function) be defined as;

$$s = e_2 + ce_1 \quad \text{..... (20)}$$

Consequently the sliding variable time derivative is

$$\dot{s} = \dot{e}_2 + c\dot{e}_1 \quad \text{..... (21)}$$

where $c > 0$ and

$e_1 = x_1 - x_d$, $\dot{e}_1 = e_2$
$e_2 = de_1 = x_2 - \dot{x}_d$, $\dot{e}_2 = \dot{x}_2 - \ddot{x}_d$

When the sliding variable reaches the zero value (the switching manifold ($s = 0$) the error

function e_1 will decay asymptotically to the origin since $c > 0$. The second step is devoted to derive the control law that will enforce the state to reach the sliding manifold in finite time.

From Eq. (18) $\dot{s}$ becomes

$$\dot{s} = f_{2o}(x) + g_{2o}(x)\vartheta + \delta(x,u) + ce_2 - \ddot{x}_d$$
$$= f_{2o}(x) + g_{2o}(x)\vartheta + \delta(x,u) + cx_2 - c\dot{x}_d - \ddot{x}_d \quad \ldots\ldots (22)$$

To design a sliding mode controller the candidate Lyapunov function is selected as

$$V = |s|$$

and its time derivative $\dot{V}$ is

$$\dot{V} = \dot{s} * sgn(s), \qquad s \neq 0$$

This is known as the generalized derivative since the candidate Lyapunov function is non-smooth [14].

or

$$\dot{V} = sgn(s) * (f_{2o}(x) + g_{2o}(x)\vartheta + \delta(x,u) + cx_2) \quad \ldots\ldots (23)$$

The ask is to select ϑ such that $\dot{V}$ is negative definite. In this work ϑ is selected as in the conventional sliding mode by

$$\vartheta = \frac{-1}{g_{2o}(x)}\begin{pmatrix} f_{2o}(x) + cx_2 + k(x) * sign(s) \\ -c\dot{x}_d - \ddot{x}_d \end{pmatrix} \quad \ldots\ldots (24)$$

Then $\dot{V}$ becomes;

$$\dot{V} = sgn(s) * (-k(x) * sign(s) + \delta(x,u))$$
$$= -k(x) + sign(s) * \delta(x,u)$$
$$\leq -k(x) + |\delta(x,u)| \quad \ldots\ldots (25)$$

The gain $k(x)$ that will make the inequality (25) less than zero (attractiveness of the sliding manifold and sliding motion)is selected as follows;

$$k(x) > |\delta(x,u)| \quad \ldots\ldots (26)$$

where

$$|\delta(x,u)| = |\Delta f_2(x)| + |\Delta g_2(x)\vartheta|$$
$$= |\Delta f_2(x)| + \left|\Delta g_2(x) * \left\{\left(\frac{-1}{g_{2o}(x)}\right)(f_{2o}(x) + cx_2 + k(x) * sign(s) - c\dot{x}_d - \ddot{x}_d)\right\}\right|$$

$$= |\Delta f_2(x)| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * |f_{2o}(x) + cx_2 - c\dot{x}_d - \ddot{x}_d| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * k(x) \quad \ldots\ldots (27)$$

Substituting Eq. (27) in the inequality (26) and solving for $k(x)$ we obtain:

$$k(x) > \frac{\max\left\{|\Delta f_2(x)| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * (|f_{2o}(x) + cx_2 - c\dot{x}_d - \ddot{x}_d|)\right\}}{1 - \max\left(\left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right|\right)}$$

or

$$k(x) = k_o + \frac{\max\left\{|\Delta f_2(x)| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * (|f_{2o}(x) + cx_2 - c\dot{x}_d - \ddot{x}_d|)\right\}}{1 - \max\left(\left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right|\right)} \quad \ldots\ldots (28)$$

where k_o is a positive constant, and

$$|\Delta f_2(x)| = \left|\Delta a_{21}\sin\left(\frac{x_1}{\gamma}\right) + \Delta a_{22}x_2\right| \leq |\Delta a_{21}||\sin\left(\frac{x_1}{\gamma}\right)| + |\Delta a_{22}||x_2|,$$

$|\Delta g_2(x)| = |\Delta a_{23}|$, and

$$(|f_{2o}(x) + cx_2 - c\dot{x}_d - \ddot{x}_d| = \left|\begin{matrix} a_{21o}\sin\left(\frac{x_1}{\gamma}\right) - a_{22o}x_2 + cx_2 \\ -c\dot{x}_d - \ddot{x}_d \end{matrix}\right|$$

$$\leq a_{21o}\left|\sin\left(\frac{x_1}{\gamma}\right)\right| + |c - a_{22o}||x_2| + c|\dot{x}_d| + |\ddot{x}_d|$$

The control law $\boldsymbol{u}$ eventually is given by:

$$u = \frac{1}{g_3}\left(-f_3(x) + \frac{1}{T}(-x_3 + \vartheta)\right) \quad \ldots\ldots (29)$$

Finally the control design idea, that is presented in this work, can be summarized here as follows;

1) Transforming the electrical subsystem via state feedback to a LPF with the desired time constant. This step (which it is a primary control design) is a continuous function of the EMS states.

2) Reducing the dynamical model of the electromechanical system by ignoring the LPF for small time constant. This step reduce the model to a mechanical system only.

3) Designing a sliding mode controller (discontinuous controller) for the mechanical system where the mismatch property is removed. The designed discontinuous controller will be able to controlling the mechanical subsystem in the presence of the uncertainty and disturbances in system model.

3.2 The Control Design and Chattering Attenuation Property:

The controller which is the input to the low pass filter (LPF) consists of a continuous primary controller and the discontinuous controller while the output of the LPF is the actual control input for the mechanical system. The output of the LPF with suitable time constant will be the equivalent control for the sliding mode controller [13]. Therefore the control system insensitivity to uncertainty and external disturbances are preserved especially after reaching sliding manifold. Moreover the chattering in system response which is directly related to the amplitude of the discontinuous control is attenuated since the actual control affected the mechanical system

will have smaller discontinuous control gain k at the sliding manifold.

4. Simulations Result and Discussion

In this section, the numerical simulations for the electromechanical system implemented are based on the full order model as given in Eqs. (10, 11, 12), with the control law from Eq. (29) and $k(x)$ as determined in Eq. (28). The simulations are performed using MATLAB with initial condition $(\theta(0), \dot{\theta}(0), i(0)) = (0,0,0)$ and the parameters for the electromechanical system as given in table (1) below. In addition the slope used for the sliding manifold is $c = 35$

Table 1: The parameters used for the electromechanical system [11]

Parameter	Definition	Value	Units
R_a	Armature resistance	0.316	Ω
L_a	Armature inductance	0.00008	H
k_m	Torque constant	0.0302	Nm/A
k_n	Induction constant of the DC motor	60/317	Vs
J	Moment of inertia of the motor load system	1.3400e-005	kg m^2
c_1	Friction constant	0.003	Nms/rad
γ	Gear reduction of the motor	91	
m	Pendulum mass	0.3	Kg
g	Gravitational constant	9.81	m/s^2
l	Pendulum length	0.5	M

Two simulation tests are presented below which aim firstly to show the ability and features of the proposed sliding controller as derived in section 3.1, and secondly to compare the obtained results with the sliding mode controller designed after ignoring the inductance in the electrical subsystem. The comparison include the time required to reach the reference value, the control input and the chattering that is induced due to the discontinuity in the control law. The classical SMC with ignored inductance is derived in Appendix A (Equation (A22) and Eq. (A26) for the control law and the gain k respectively).

Test one: Reference angle is constant ($\theta_r = 0.175$ rad)

Figure (2) shows the simulation result for the angle θ with time and with variation in moment of inertia equal to (20%). Plotting the sliding variable $s(t)$ demonstrate the effectiveness and the robustness of the sliding mode controller where the system dynamics is free from the uncertainty in moment of inertia when $s(t)$ becomes equal to zero after $(0.2\ second)$. The sliding variable is plotted versus time in figure (3) with a small bound of oscillation around the switching manifold. In figure (4), the control action $u(t)$ is plotted for the electromechanical system using the proposed sliding mode controller. In addition figure (5) shows the phase plot of $x_2 = \dot{\theta}$ vs. $x_1 = \theta$.

Figures (6)-(9) summaries the simulation results for the electromechanical system with ignored(negligible) inductance of the DC motor (L≈0). Figure (6) shows the simulation result for the angle θ versus time and with variation in load equal to (20%) while figure (7) plots the sliding variable $s(x)$ with time. The time required to reach the sliding manifold does not exceed $0.01\ second$. The control action $u(t)$ is plotted for the electromechanical system control system with ignored inductance (L) in figure (8). Figure (9) shows the phase plot of $x_2 = \dot{\theta}$ *vs.* $x_1 = \theta$. Finally figures (10) and (11) summarize the phase plot of $x_2 = \dot{\theta}$ *vs.* $x_1 = \theta$ and the control effort $u(t)$ versus time for the electromechanical system with and without ignored inductance (L).

As mentioned previously, two features of the present proposed SMC can be distinguished from figures (2)-(9) which are; chattering attenuation and the magnitude of the control effort. Chattering attenuation is well clarified for the proposed controller when comparing Figs. (3) and (7) and it can also be deduced from the plot of the control input in Figs. (4) and (8), where a high oscillation is shown in Fig. (8) for the case of ignoring inductance. As a final proof for chattering attenuation property in the proposed controller is the phase plane plot and the control effort for the proposed and the classical SMC plotted in Figs. (10) and (11) respectively. In these figures the amplitude of oscillation around an average value can be taken as a measure for state chattering. Figure (10) plot the amplitude of the state oscillation around the sliding manifold which it is greatly reduced when using the present proposed controller. In addition, the attenuation property can be deduced from Fig. (11) where the control input voltage highly oscillated around an average value represent by the control effort using the proposed control law. The control voltage according to the proposed control law is nearly equal the equivalent control [13] (it also named "the real sliding mode control" [15]) where the sliding motion is preserved with less chattering amplitude. The control input (the voltage) required in this design is $0.77\ Volt$ while in the case of ignoring inductance its value is $2.2\ Volt$ (approximately three times) (Fig. (11)).

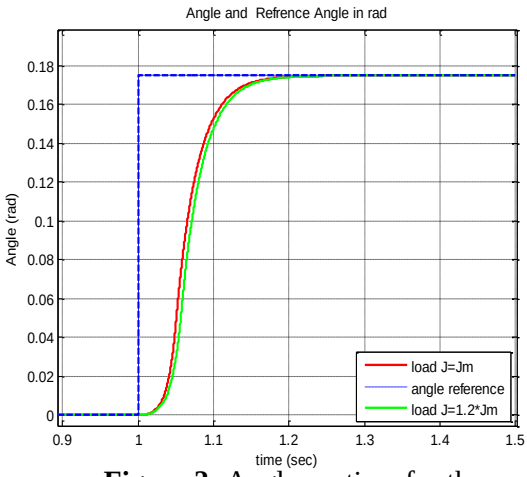

Figure 2: Angle vs. time for the Electromechanical System

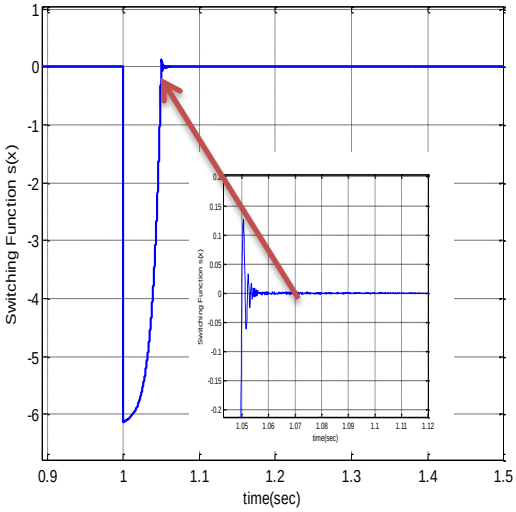

Figure 3: Sliding Variable s(x) vs. time for the Electromechanical System

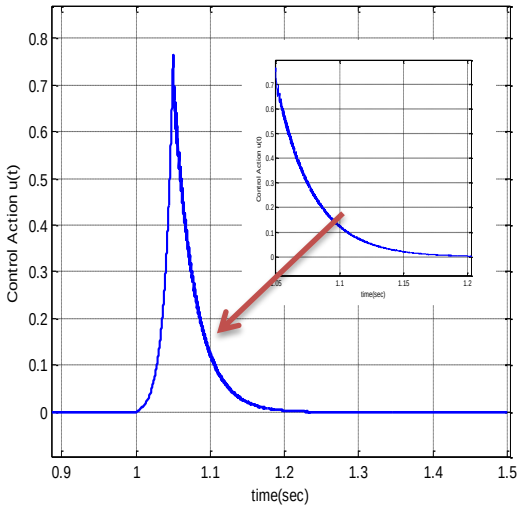

Figure 4: Control Action u(t) vs. time for the Electromechanical System

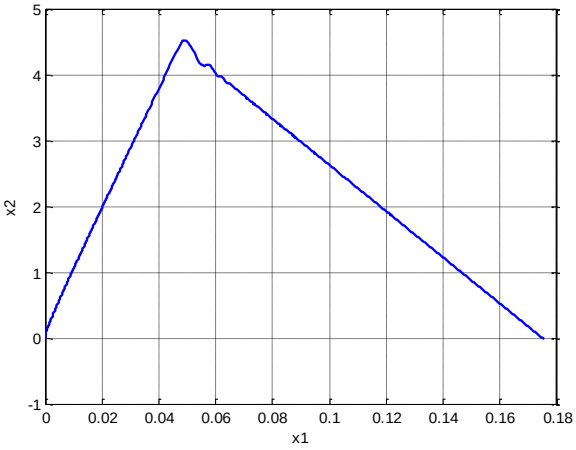

Figure 5: phase plot of x_2 vs. x_1 for the Electromechanical System

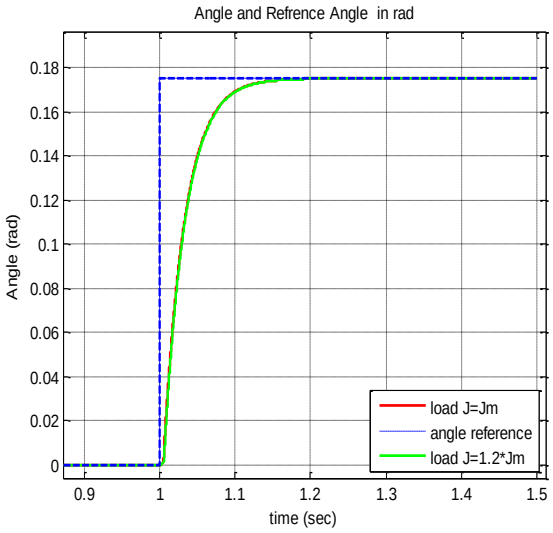

Figure 6: Angle vs. time for the Electromechanical System with ignored

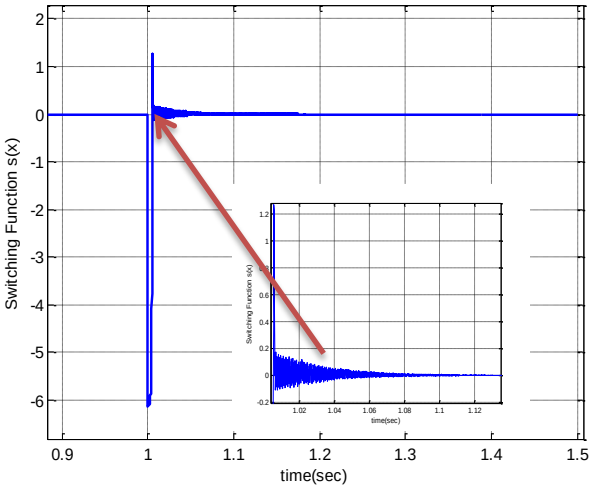

Figure 7: Sliding Variable s(x) vs. time For the Electromechanical System with ignored inductance (L)

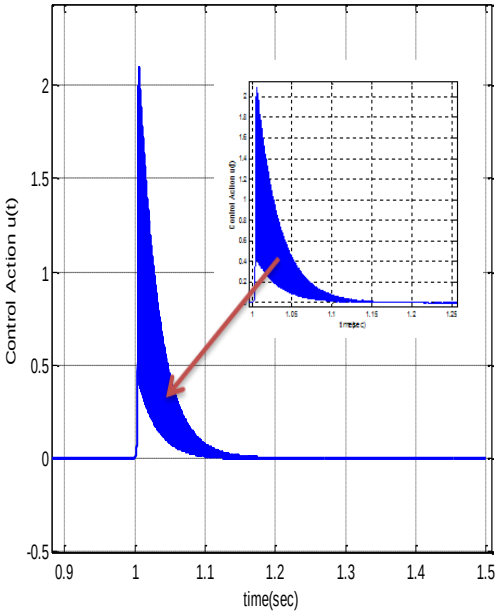

Figure **8:** Control Action u(t) vs. time for the Electromechanical System with ignored inductance (L)

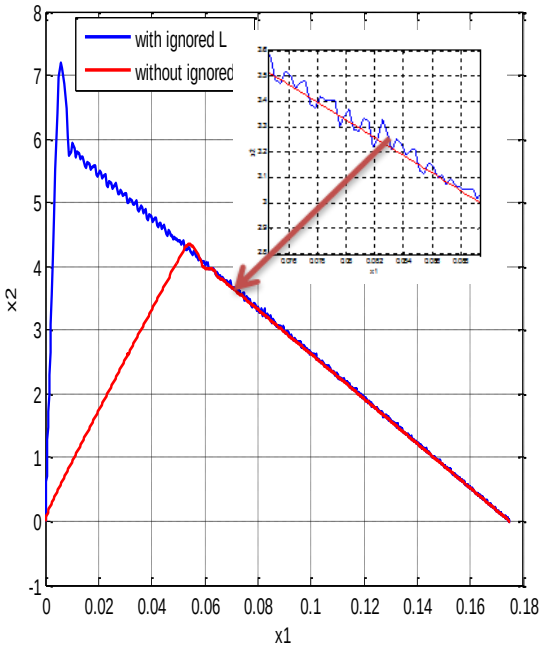

Figure 10: phase plot of x_2 vs. x_1 for the Electromechanical System with and without ignored inductance (L)

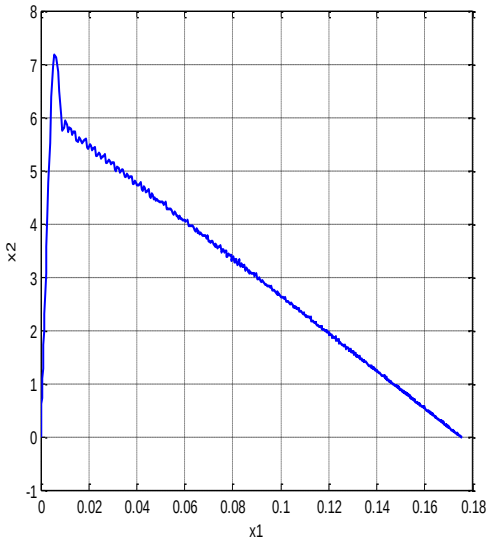

Figure 9: phase plot of x_2 vs. x_1 for the Electromechanical System with ignored inductance (L)

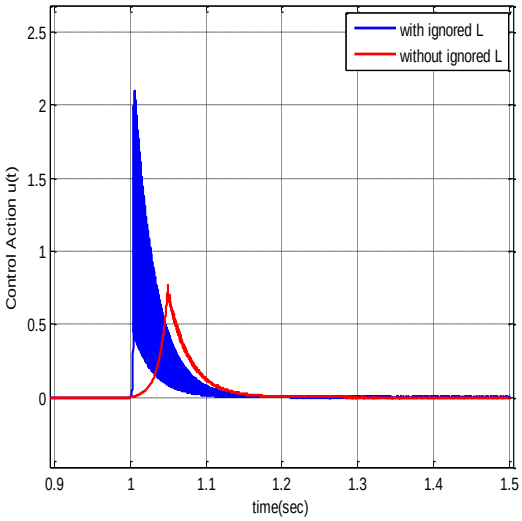

Figure 11: Control Action u(t) vs. time for the Electromechanical System with and without ignored inductance (L)

Test two: Reference angle is piecewise constant

$\theta_r = 0$ rad. $for \quad 0 \le t \le 1$

$= 0.175$ rad. $for \quad 1 < t \le 1.5$

$= 0 \quad$ rad. $for \quad 1.5 < t \le 2$

$= 0.175$ rad. $for \quad 2 < t \le 2.5$

$= 0$ rad. $for \quad 2.5 < t \le 3$

The ability and the effectiveness of the proposed controller is again tested for a piecewise constant reference angle given above. The

simulation results are plotted in Fig. (12) to show the ability of the SMC in forcing the angle θ to follow the desired trajectory with the variation in load equal to (20%) while the sliding variable $s(x)$ and the control action $u(t)$ are plotted with time in Figs. (13) and (14) respectively. As can be seen from Fig. (13) the amplitude of state oscillation around the sliding manifold is small and consequently the chattering is attenuated.

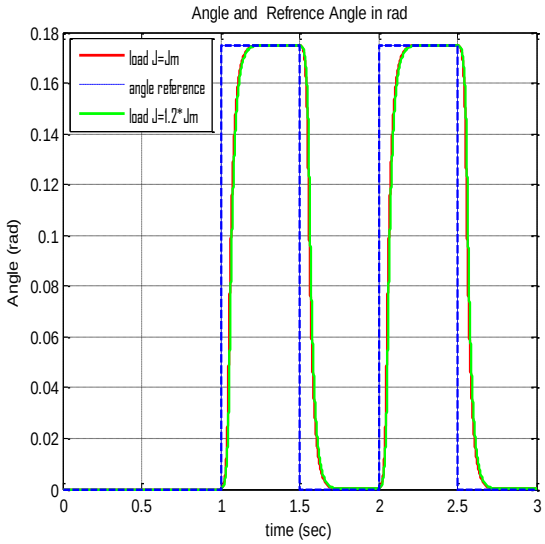

Figure 12: Angle vs. time for the Electromechanical System

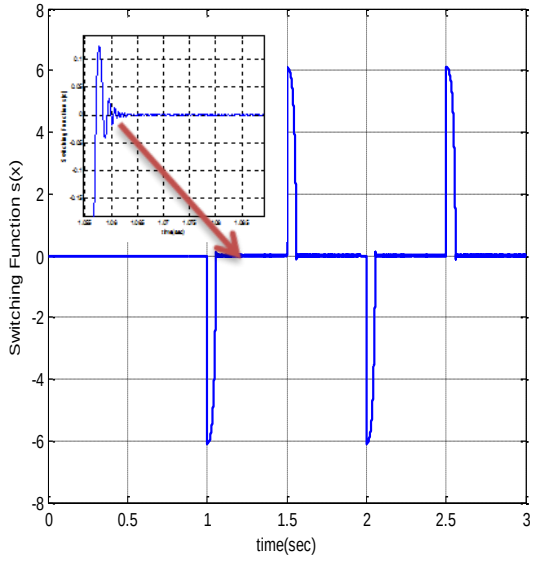

Figure 13: Sliding Variable s(x) vs. time For the Electromechanical System

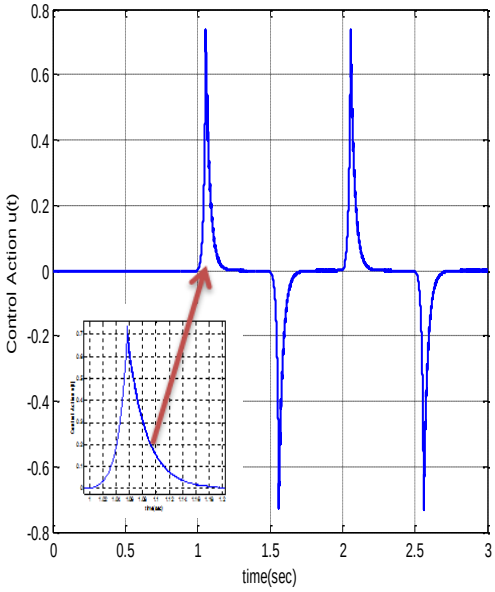

Figure 14: Control Action u(x) vs. time For the Electromechanical System

5. Conclusions

In this paper a sliding mode control for electromechanical systems was developed with new approach. Results show that the proposed control scheme improves performance of electromechanical systems compared to conventional control schemes. Namely the proposed method consists of transforming the electrical subsystem to a Low Pass Filter with a suitable time constant allows, then designing a sliding mode controller to the mechanical subsystem in the usual way. This approach avoiding neglecting the electrical part (ES) or approximation the non-smooth perturbation and achieving the matching condition. As results the chattering is attenuated with smaller control efforts (the voltage). The simulation results prove first the robustness and effectiveness of the proposed controller for a bounded uncertainty in system parameters with the presence of nonlinearity. Secondly the simulation results, for the reference pendulum angle $\theta_r = 0.175$ rad, show that the chattering is attenuated when compared with design sliding mode control of electromechanical system with ignoring the electrical part. In addition the control effort is reduced approximately three times the control efforts value required for the SMC with ignoring the electrical part.

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Appendix (A): SMC design with ignoring inductance

The electromechanical system model:

$$\dot{x}_1 = x_2 \quad \text{(A.1)}$$

$$\dot{x}_2 = \frac{1}{J}(k_m x_3 - c_1 x_2 + \frac{1}{\gamma} mgl \sin(\frac{x_1}{\gamma})) \quad \text{(A.2)}$$

$$\dot{x}_3 = \frac{1}{L}(-R_a x_3 - k_n x_2 + u) \quad \text{(A.3)}$$

To simplify the electromechanical system model, the motor inductance L is ignored (i.e., by considering $L \approx 0$). Accordingly the electromechanical system model in Equations (A.1-A.3) are reduced to a system with lower dimension as can be noted in the following steps:

$$L\dot{x}_3 = (u - R_a x_3 - k_n x_{2)} \quad \text{(A.4)}$$

For $L \approx 0$

$$0 \approx -R_a x_3 - k_n x_2 + u \quad \text{(A.5)}$$

$$R_a x_3 = -k_n x_2 + u \quad \text{(A.6)}$$

Now solving for x_3 yields:

$$x_3 = \frac{-k_n}{R_a} x_2 + \frac{1}{R_a} u \quad \text{(A.7)}$$

Substitution the value of x_3 from Equation (A.7) into Equation (A.2), yields:

$$\dot{x}_1 = x_2 \quad \text{(A.8)}$$

$$\dot{x}_2 = \frac{1}{J}(k_m(\frac{-k_n}{R_a} x_2 + \frac{1}{R_a} u) - c_1 x_2 + \frac{1}{\gamma} mgl \sin(\frac{x_1}{\gamma})) \quad \text{(A.9)}$$

The system can be arranged as follows

$$\dot{x}_1 = x_2 \quad \text{(A.10)}$$

$$\dot{x}_2 = \frac{1}{J\gamma} mgl \sin\left(\frac{x_1}{\gamma}\right) - (\frac{c_1}{J} - \frac{k_m k_n}{JR_a}) x_2 + \frac{1}{JR_a} u \quad \text{(A.11)}$$

Let:

$$a_{21} = \frac{1}{J\gamma} mgl \,,\ a_{22} = \frac{c_1}{J} + \frac{k_m k_n}{JR_a} \,,\ b_2 = \frac{1}{JR_a}$$

The reduced electromechanical system can be written as:

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= a_{21}\sin\left(\frac{x_1}{\gamma}\right) - a_{22}x_2 + b_2u\end{aligned}\right\} \quad (A.12)$$

Let $f_2(x) = a_{21}\sin\left(\frac{x_1}{\gamma}\right) - a_{22}x_2$

$g_2(x) = b_2$

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f_2(x) + g_2u\end{aligned}\right\} \quad (A.13)$$

Writing Equation (A.13) as a nominal and perturbation term, yields;

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f_{2o}(x) + g_{2o}u + \delta(x,u)\end{aligned}\right\} \quad (A.14)$$

where $f_{2o}(x)$ and $g_{2o}(x)$ are the nominal functions of $f_2(x)$ and $g_2(x)$ respectively while $\delta(x,u)$ is the uncertainty term results from the uncertainty in system dynamics. $\delta(x,u)$ is given by

$$\delta(x,u) = \Delta f_2(x) + \Delta g_2(x)u \quad (A.15)$$

Let the sliding variable be defined as;

$$s = e_2 + ce_1 \quad (A.16)$$

Consequently the sliding variable time derivative is

$$\dot{s} = \dot{e}_2 + c\dot{e}_1 \quad (A.17)$$

where

$e_1 = x_1 - x_d$, $\dot{e}_1 = e_2$

$e_2 = de_1 = x_2 - \dot{x}_d$, $\dot{e}_2 = \dot{x}_2 - \ddot{x}_d$

$$\dot{s} = \dot{e}_2 + c\dot{e}_1 = \dot{x}_2 - \ddot{x}_d + c(x_2 - \dot{x}_d)$$

From Eq. (A.14) $\dot{s}$ becomes

$$\dot{s} = f_{2o}(x) + g_{2o}(x)u + \delta(x,u) + cx_2 - c\dot{x}_d - \ddot{x}_d \quad (A.18)$$

To design a sliding mode controller the candidate Lyapunov function is selected as

$$V = |s| \quad (A.19)$$

and its time derivative $\dot{V}$ is

$$\dot{V} = \dot{s} * sign(s),\ s \neq 0 \quad (A.20)$$

or

$$\dot{V} = sign(s) * (f_{2o}(x) + g_{2o}(x)u + \delta(x,u) + cx_2 - c\dot{x}_d - \ddot{x}_d) \quad (A.21)$$

The ask is to select u such that $\dot{V}$ is negative definite. In this work u is selected as in the conventional sliding mode by

$$u = \frac{1}{g_{2o}(x)}\left(-f_{2o}(x) - cx_2 - c\dot{x}_d - \ddot{x}_d - k(x) * sign(s)\right) \quad (A.22)$$

Then $\dot{V}$ becomes;

$$\begin{aligned}\dot{V} &= sign(s) * (-k(x) * sign(s) + \delta(x,u))\\ &= -k(x) + sign(s) * \delta(x,u)\\ &\leq -k(x) * +|\delta(x,u)|\end{aligned} \quad (A.23)$$

The gain $k(x)$ that will make the inequality (A-23) less than zero (attractiveness of the sliding manifold and sliding motion) is selected as follows;

$$k(x) > |\delta(x,u)| \quad (A.24)$$

where

$$\begin{aligned}&|\delta(x,u)| = |\Delta f_2(x)| + |\Delta g_2(x)u|\\ &=|\Delta f_2(x)| + \left|\Delta g_2(x) * \left\{\left(\frac{1}{g_{2o}(x)}\right)(-f_{2o}(x) - cx_2 - c\dot{x}_d - \ddot{x}_d - k(x) * sign(s))\right\}\right|\\ &=|\Delta f_2(x)| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * |f_{2o}(x) - cx_2 - c\dot{x}_d - \ddot{x}_d| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * k(x)\end{aligned} \quad (A.25)$$

By substitute Eq. (A.25) in the inequality (A.24) and solving for $k(x)$ we obtain:

$$k(x) > \frac{\max\left\{|\Delta f_2(x)| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * (|f_{2o}(x)| + c|x_2| + c|\dot{x}_d| + |\ddot{x}_d|)\right\}}{1 - \max\left(\left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right|\right)}$$

or

$$k(x) = k_o + \frac{\max\left\{|\Delta f_2(x)| + \left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right| * (|f_{2o}(x)| + c|x_2| + c|\dot{x}_d| + |\ddot{x}_d|)\right\}}{1 - \max\left(\left|\frac{\Delta g_2(x)}{g_{2o}(x)}\right|\right)} \quad (A.26)$$

where k_o is a positive constant and

$$\begin{aligned}&|\Delta f_2(x)| = \left|\Delta a_{21}\sin\left(\frac{x_1}{\gamma}\right) + \Delta a_{22}x_2\right|\\ &\leq |\Delta a_{21}||\sin\left(\frac{x1}{\gamma}\right)| + |\Delta a_{22}||x_2|,\\ &|\Delta g_2(x)| = |\Delta b_2|,\\ &|f_{2o}(x) - cx_2 - c\dot{x}_d - \ddot{x}_d| = \left|a_{21o}\sin\left(\frac{x_1}{\gamma}\right) - a_{22o}x_2 - cx_2 - c\dot{x}_d - \ddot{x}_d\right|\\ &\leq |a_{21o}|\left|\sin\left(\frac{x_1}{\gamma}\right)\right| + |a_{22o} + c||x_2| + c|\dot{x}_d| + |\ddot{x}_d|\end{aligned}$$

مسيطر ذو شكل إنزلاقي للأنظمة الكهروميكانيكية مع تخفيف للإرتجاج

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الخلاصة:

يمكن إعتبار الأنظمة الكهروميكانيكية كآلات تقوم بتحوبل الطاقة الكهربائية الى طاقة ميكانيكية. كل نظام كهروميكانيكي يمكن تجزئته الى نظام جزئي كهربائي (ES) و نظام جزئي ميكانيكي (MS). بسبب عوامل كثيرة قد تكون السيطرة على حركة هذه الأنظمة غاية بالتعقيد حيث يجب الأخذ بهذه العوامل عند التصميم. يمكن تلخيص هذه العوامل على إنها اللاخطية، الخشونة (non-smoothness) في النموذج الرياضي، الشك أو عدم اليقين في معاملات النموذج وأيظا عدم تحقيق النموذج شرط الموائمة (the matching condition).

في هذا البحث تم إقتراح إسلوب جديد في تصميم المسيطر المنزلق للأنظمة الكهروميكانيكية بدون الحاجة لإلغاء المحاثة في الدائرة الكهربائية أو تقريب التشويش الخشن. تتمثل الخطوة الأولى بتحوبل النظام الكهربائي الى مرشح منخفض (LPF) بينما الخطوة الثانية تتمثل بتصميم مسيطر منزلق للنظام الميكانيكي والتي ستتمكن من إزالة تأثير الشك و الخشونة بالنموذج الرياضي. بإختيار مناسب للثابت الزمني لل (LPF) يكون المسيطر المنزلق هو المسيطر المكافئ وكنتيجة لذلك ستخفف الإرتجاجات بشكل أكبر من تلك في حالة التصميم التقليدي للمسيطر المنزلق والذي صمم بعد إهمال النظام الكهربائي وأيضا مع فولتية داخلة أقل. إن نتائج المحاكات الرياضية لمنظومة كهروميكانيكية معينة أظهرت علو مقدرة التصميم المفترح بالمقارنة بالتصميم التقليدي وبخاصيتين مهمتين للمسيطر المنزلق ،بجانب قدرته على إرغام متغير الحالة لتتبع موقع معين، حيت تم تخيض الإرتجاج بشكل كبير وأيظا تقليل الفولتية الداخلة (تقريبا ثلث الفولتية المطلوبة في حالة امسيطر المنزلق التقليدي).