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Flexure Revisited: Strength of Singly Reinforced Beams- A Simple Approach

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Abstract

181 singly reinforced beam tests available from the literature are investigated to obtain two flexural design methods-one simple proposal and an alternative one that takes into account the influence of rising flexural reinforcement ratio on design. The simple proposal takes significantly less time to apply compared to the alternative one. The former leads to only 0.5% greater COV than the latter. These design methods are compared with four code methods (ACI 318M-11, 318M-99, BS and NZ codes).

Based on the ratio of M_{test}/M_r (M_r being the calculated moment resistance), the two proposal methods give relatively low coefficient of variation (COV) values: 15.9% for the simple method and 15.4% for the alternative method. These compare to COV values of 17.5%, 15.9%, 15.6% and 15.7% for the ACI 318M-11, ACI 318M-99, BS and NZ methods, respectively.

One major advantage of the simple design method (Constant value of ϕ = 0.85) is that all 181 tests lead to safe prediction.

Keywords: beams; flexure; nominal strength; reinforced concrete; ultimate strength.

Introduction

The most recent ACI code (318M-11) [1] treats beams in flexure with varying values of the strength reduction factor ϕ. This is basically modified from the first code in 2002 [2] which relates ϕ to the difference between tension control, compression control and transition zone. Reference 1 relates ϕ to **Fig.1**.

This trend contrasts with previous ACI code design (1999 code [3] and editions prior to 1999) where ϕ has a constant value of 0.9. Other codes [4,5] have a different approach to RC beam flexural design. These codes also have simple design approaches to flexural design of RC beams, in a similar manner to reference 3.

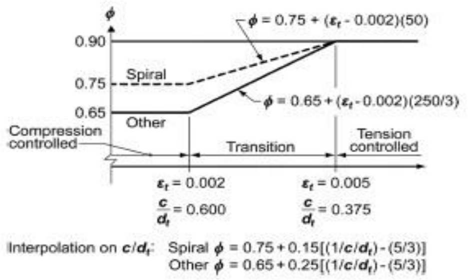

Figure.1: Variation of ϕ with net tensile strain in extreme tension steel, ε_t, and c/d_t- ACI 318M-11 [1].

In this work it is intended to apply design in flexure for singly reinforced beams, using experimental data from 181 tests published in the literature [6-28]. **Table 1** indicates the ranges of values for the variables in these beams.

Table 1: Details of 181 beam tests [6-28]

Variable	Range
f'_c, MPa (psi)	8.6-110 (1244-15915)
ρ	0.0021-0.0684
b/d	0.425-1.531
c/d	0.031-0.684

*All specimens were tested at 28 days.

Research Significance

A simplified design of RC beams in flexure is introduced, which will be related to 181 test data of singly reinforced beams failing in flexure. This will be compared to other simple code design methods [3-5]. In addition, design by ACI 318M-11 [1] is also included in the comparison.

An alternative modification to reference 1 is also studied and included in this work, in an effort to reduce the COV of the ratio of tested/calculated moment capacity.

Existing Design Methods

The following includes brief details of design based on 4 code methods:

1. ACI 318M-11 [1] Code design

Essentially this design for singly reinforced beams is based on the details of **Fig.1**, in addition to **Fig.2**.

$$a = \frac{A_s . f_y}{0.85\, f_c' . b} \qquad .....(1)$$

$$c = \frac{a}{\beta_1} \quad \text{.....(2)}$$

$$\beta_1 = \begin{cases} 0.85 & f_c' \le 28 \text{ MPa} \\ 0.85 - \frac{0.05}{7}(f_c' - 28) & 28 \text{ MPa} < f_c' < 56 \text{ MPa} \\ 0.65 & f_c' \ge 56 \text{ MPa} \end{cases} \quad \text{.....(3)}$$

$$M_n = A_s . f_y \left(d - \frac{a}{2}\right) \quad \text{.....(4)}$$

$$M_u [= M_r] = \phi M_n \quad \text{.....(5)}$$

$$\phi = \begin{cases} 0.9 & \frac{c}{d_t} \le 0.375 \\ 0.65 + 0.25\left[\frac{1}{\frac{c}{d_t}} - \frac{5}{3}\right] & 0.375 < \frac{c}{d_t} < 0.6 \\ 0.65 & \frac{c}{d_t} \ge 0.6 \end{cases} \quad \text{.....(6)}$$

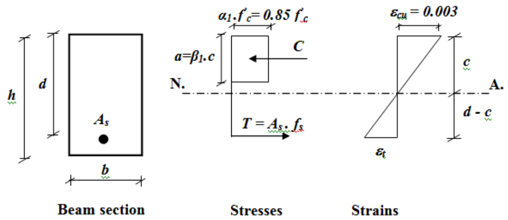

Figure.2: Ultimate stress and strain distribution at the cross section ACI 318M [1-3].

2. ACI 318M-99 [3] Code design

This design is essentially identical with reference 1, except for using a constant value of $\phi = 0.9$.

3. BS 8110: 1997 [4] Code design

Design is based on **Fig.3**.

$$a = \frac{0.95\, A_s . f_y}{0.5625\, f_c' . b} \quad \text{.....(7)}$$

Where f_c' is used per Eq. (8):

$$f_c' = 0.8\, f_{cu} \quad \text{.....(8)}$$

$$M_r = 0.95\, A_s . f_y \left(d - \frac{a}{2}\right) \quad \text{.....(9)}$$

4. New Zealand [5] Code design

Design is based on **Fig.2**, except for the modification of Eq. (10) for α_1:

$$\alpha_1 = \begin{cases} 0.85 & f_c' \le 55 \text{ MPa} \\ 0.85 - 0.004(f_c' - 55) & 55 \text{ MPa} < f_c' < 80 \text{ MPa} \\ 0.75 & f_c' \ge 80 \text{ MPa} \end{cases} \quad \text{.....(10)}$$

$$a = \frac{A_s . f_y}{\alpha_1 . f_c' . b} \quad \text{.....(11)}$$

Where $M_u [= M_r]$ is per Eq. (6); using a constant value of $\phi = 0.9$.

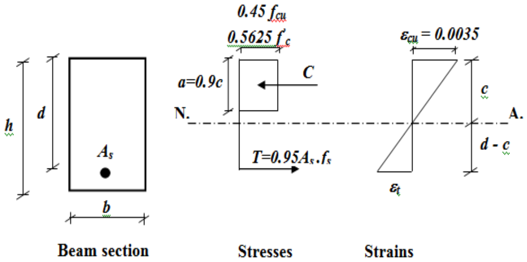

Figure.3: Ultimate stress and strain distribution at the cross section-BS 8110 [4].

Proposed Simple Design

Similarly to other simple code design methods [3-5], Eq. (12) is proposed for flexural design of singly reinforced concrete beams:

$$M_r = 0.85\, A_s . f_y \left(d - \frac{a}{2}\right) \quad \text{.....(12)}$$

Eq. (12) is a result of regression analysis, including all details, leading to the lowest possible COV. In this equation, (*a*) is applied per Eq. (1), which agrees with reference 1 (ACI 318M-11 Code).

Alternative Design Method

To acknowledge the influence on ductility of lower steel ratios, the following procedure is proposed as an alternative design for reinforced beams in flexure. This method uses the details indicated in **Fig.2**.

Eq. (13) gives the influence of f_c' on α_1 and β_1:

$$\alpha_1 = \beta_1 = \begin{cases} 0.95 & f_c' \le 22 \text{ MPa} \\ 0.95 - \frac{0.05}{7}(f_c' - 22) & 22 \text{ MPa} < f_c' < 50 \text{ MPa} \\ 0.75 & f_c' \ge 50 \text{ MPa} \end{cases} \quad \text{.....(13)}$$

This alternative design equation is identical with Eq. (6), where ϕ follows Eq. (14) and the details of **Fig.4**:

$$\phi = \begin{cases} 0.9 & c/d_t \le 0.375 \\ 0.75 + 0.15\left[\frac{1}{c/d_t} - \frac{5}{3}\right] & 0.375 < c/d_t < 0.6 \\ 0.75 & c/d_t \ge 0.6 \end{cases} \quad \text{.....(14)}$$

Eqs. (13) & (14) are intended to achieve the lowest COV values for the alternative method, based on regression analysis.

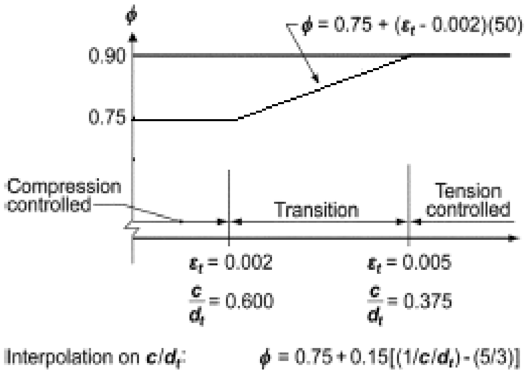

Figure.4: Variation of ϕ with net tensile strain in extreme tension steel, ε_t, and c/d_t for the alternative design proposal.

Comparison of Results and Discussion

The 6 methods compared in this work have quite different approaches to predicting M_r for singly reinforced beams. Following are some comments on the studies made in this work:

17 of the 181 test results have a higher ρ than ρ_b; the latter as defined by reference 1:

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y}\left(\frac{600}{600+\ f_y}\right) \ldots..(15)$$

In all cases of comparison for these 17 beams ($\rho > \rho_b$, per ACI 318M-11 [1]) the strength M_r is based on Eq. (16):

$$M_{r-ACI-11} = \phi A_s\,.\,f_s\left(d - \frac{a}{2}\right) \ldots..(16)$$

Where:

$$a = \frac{A_s.\,f_s}{0.85\,f_c'b} \qquad \ldots..(17)$$

Eq. (17) is applied to all cases when $f_s < f_y$. **Table 2** gives a comparison of the 6 methods studied in this work: The highest COV is due to the present ACI code [1] at 17.5 percent. This contrasts with significantly lower COV values by other methods, ranging between 15.4% - 15.9%. Thus applying reference 1 design raises the COV between 10.1 and 13.6 percent, compared to the other methods. In contrast with the proposed simplified method, all other methods lead to between 3-7 unsafe predictions- the former leads to safe predictions for all 181 tests, **Table 2**.

Table 2: Statistical analysis of the ratio of (M_{test}/M_r) for 181 beam tests

Detail	ACI-11 [1]	ACI-99 [3]	BS-97 [4]	NZ-9 5[5]	Simple method	Alternative method
$\bar{x}$	1.348	1.268	1.290	1.272	1.343	1.288
S.D.	0.236	0.202	0.201	0.199	0.214	0.198
COV%	17.499	15.915	15.603	15.670	15.903	15.374
Low	0.954	0.954	0.935	0.954	1.010	0.946
High	2.326	2.315	2.204	2.319	2.451	2.319
High/Low	2.439	2.427	2.357	2.431	2.427	2.450
Number<1	4	7	3	4	0	5

Because of the significantly different approaches in the 6 methods being applied in this work, different failure modes are accommodated in these methods, regardless of whether failure is in reinforcement (tensile) or concrete (crushing). **Figs. 5-7** show a comparison between the 6 considered methods of design in flexure. Based on the effect of f_c' on the relative safety, there is a trend for slightly lower safety factor with rising f_c' in all methods, except the alternative proposal for modified ACI-11 method. In contrast to all other methods, the drop of safety is more significant with rising f_c', when the BS method is used.

All methods, except for design per ACI 318M-11 [1], lead to a drop in the safety factor as $\rho(= A_s/bd\ )$ rises. Despite this difference between reference 1 design and the proposed simplified design method, the latter leads to safe results for all 181 tests. This clearly stands out as an added advantage of the proposed simplified design method.

For the ratio of b/d between 0.425-1.531, there seems to be no significant difference between the 6 methods.

Conclusions

Based on 181 tests of singly reinforced beams failing in flexure, a study has been made on the prediction of strength, based on 6 methods – 4 code methods, plus 2 alternative ones: A simplified approach and one modified from the latest ACI code [1]. The following conclusions are drawn:

1. Essentially all 6 design methods are safe, with the ratio of M_{test}/M_r being less than one between 0 – 7 cases only out of 181

tests. The only method with no unsafe prediction is the proposed simplified method, where all ratios of M_{test}/M_r are greater than 1 for the 181 tests - as evidenced in **Table 2.**

2. The two methods presented in this work contrast with the present ACI code method in their COV values. While reference 1 has the highest COV of all 6 methods of 17.5%, the two presented methods lead to only 15.4% and 15.9% for the COV of the ratio of M_{test}/M_r.
3. The proposed alternative modification of reference 1 shows no effect on safety of prediction with rising f_c'. In contrast, BS design leads to a significant drop in safety of prediction with rising f_c'.
4. Five of the 6 methods lead to a drop with the safety factor as $\rho(= A_s/bd)$ rises, in contrast with ACI 318M-11 [1] code, which has no drop in safety. While the proposed design method (with fixed $\phi = 0.85$) has a drop in safety of prediction, this method leads to safe prediction in all 181 tests.
5. With a significant range of *b/d* (0.425 to 1.531), all 6 methods show no change in safety with the value of *b/d*.

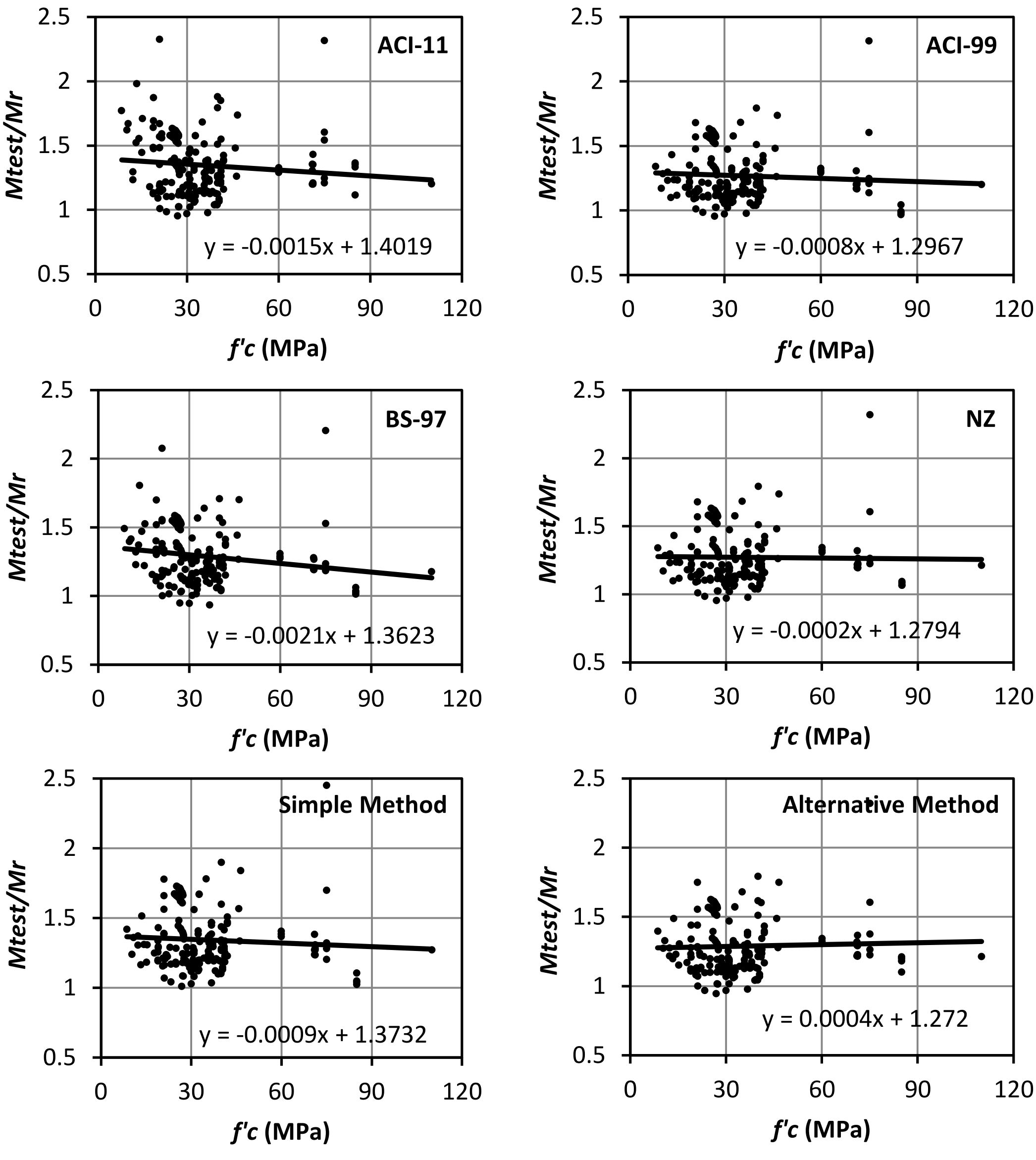

Figure.5: Influence of compressive strength of concrete f_c' on M_{test}/M_r ratio.

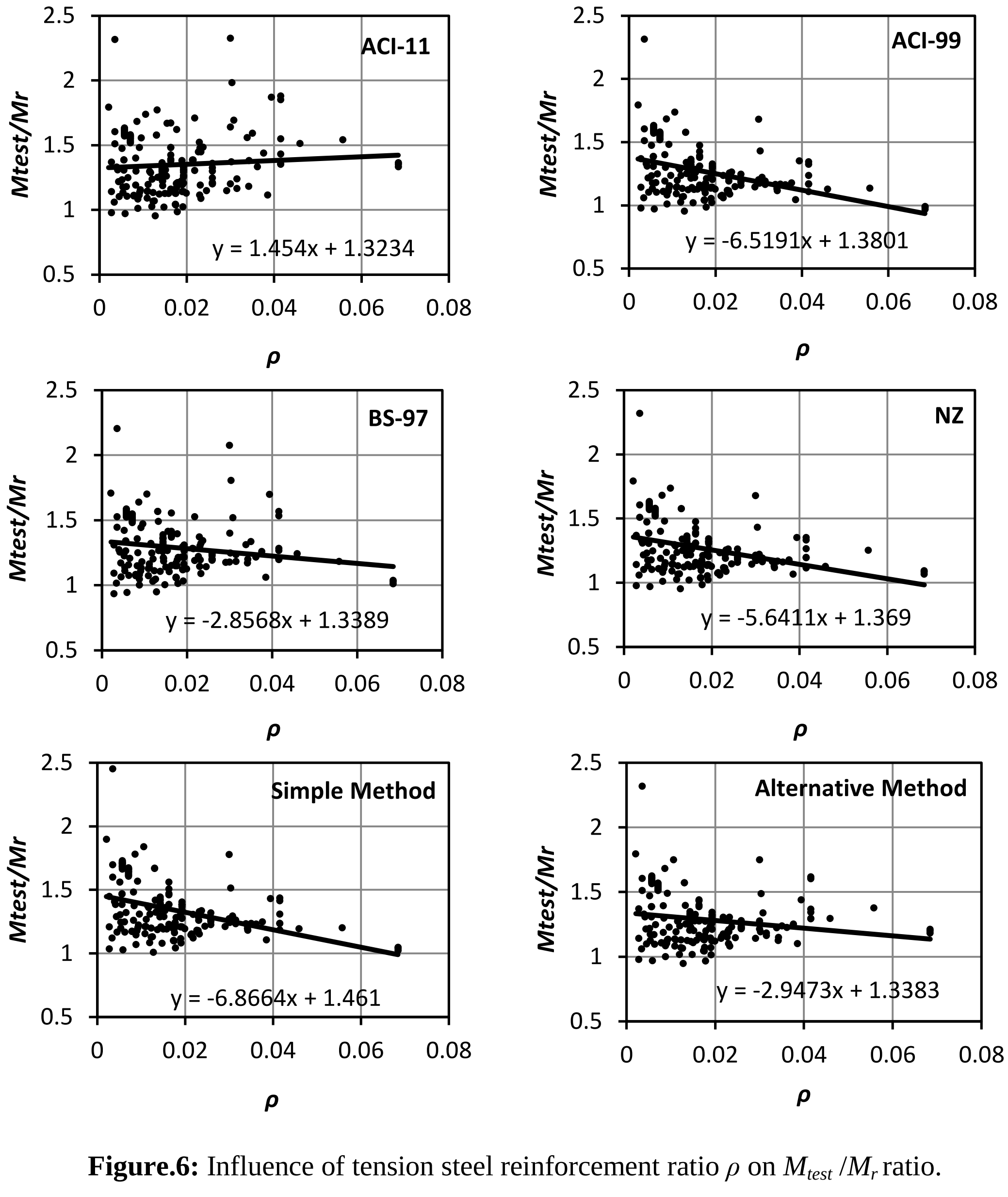

Figure.6: Influence of tension steel reinforcement ratio ρ on M_{test}/M_r ratio.

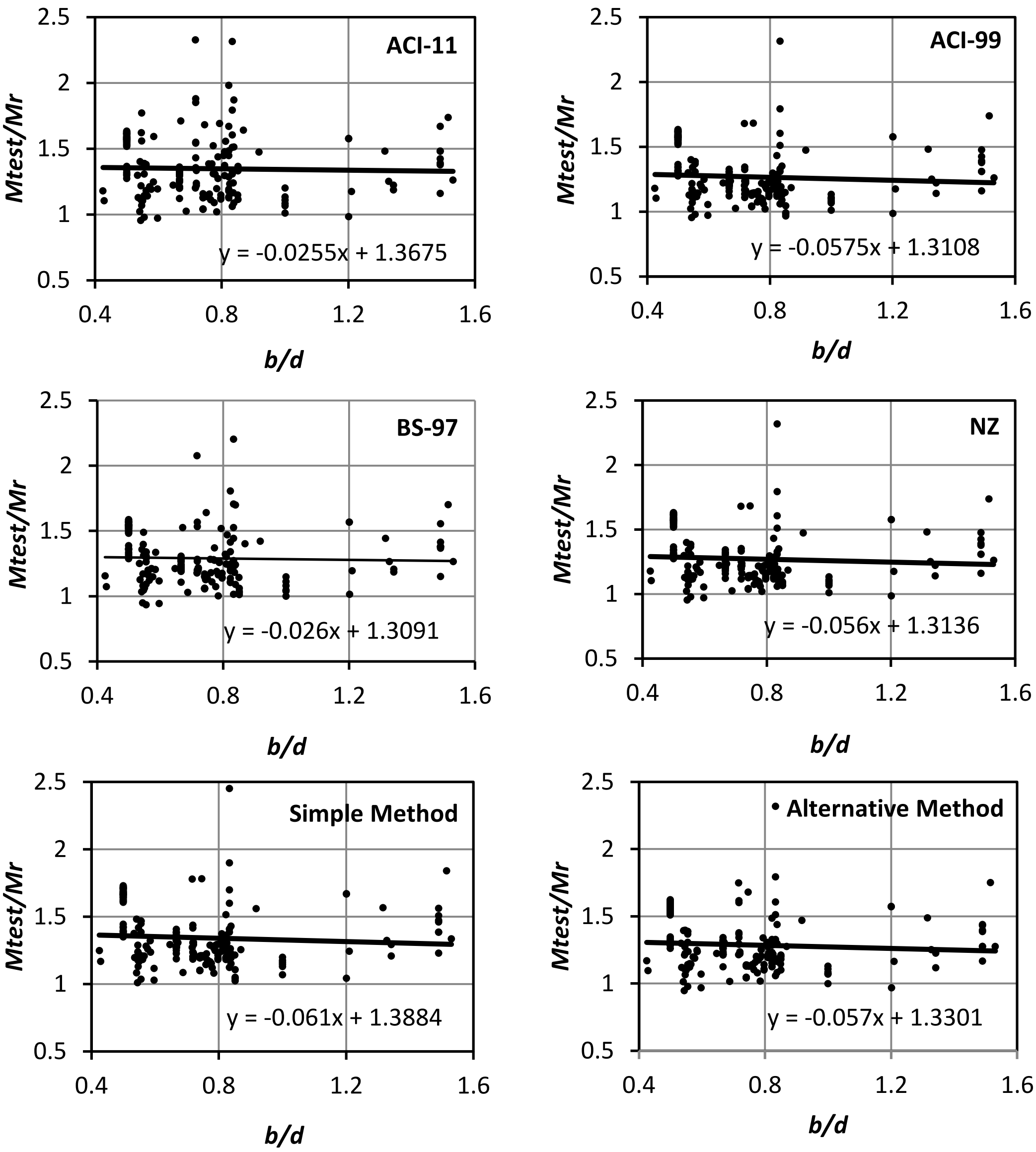

Figure.7: Influence of beam width to effective depth ratio *b/d* on M_{test}/M_r ratio.

Notation

a = depth of equivalent rectangular stress block, mm

$\boldsymbol{A_s}$ = area of nonprestressed longitudinal tension reinforcement, mm^2

b = width of compression face of member, mm

c = distance from extreme compression fiber to neutral axis, mm

C = compression force in concrete, N

COV = coefficient of variation of the ratio of M_{test}/M_r

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, mm

$\boldsymbol{d_t}$ = distance from extreme compression fiber to centroid of extreme layer of longitudinal tension steel, mm

$\boldsymbol{f'_c}$ = specified cylinder compressive strength of concrete, MPa

$\boldsymbol{f_{cu}}$ = specified cube compressive strength of concrete, MPa

f_s = calculated tensile stress in longitudinal tension steel, MPa
f_y = specified yield strength of reinforcement, MPa
h = overall thickness or height of member, mm
M_n = nominal flexural strength at section, N.mm
M_r = calculated moment resistance at section, N.mm
M_{test} = tested moment resistance at section, N.mm
M_u = factored moment at section, N.mm
S.D. = standard deviation of the ratio of M_{test} /M_r
T = tension force in longitudinal tension steel, N
$\bar{x}$ = arithmetic mean of the ratio of M_{test} /M_r
α_1 = factor related to f_c', e.g. α_1 = 0.85 for ACI design [1-3]
β_1= factor relating depth of equivalent rectangular compression stress block to neutral axis depth
ϵ_{cu} = maximum usable strain at extreme concrete compression fiber
ϵ_t = net tensile strain in extreme layer of longitudinal tension steel at nominal strength
ρ = ratio of A_s to bd
ρ_b = ratio of A_s to bd producing balanced strain Conditions [1-3]
ϕ = strength reduction factor

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مقاومة الانثناء للعتبات الخرسانية أحادية التسليح- طريقة مبسطة

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الخلاصة:
تم دراسة نتائج فحوصات 181 عتبة خرسانية أحادية التسليح مأخوذة من بحوث سابقة وذلك للحصول على طريقتين لتصميم الانثناء، الاولى طريقة مقترحة بسيطة، اما الثانية فتأخذ بنظر الاعتبار تأثير زيادة نسبة حديد تسليح الانثناء على التصميم. ان تطبيق الطريقة المبسطة يحتاج الى وقت اقصر مقارنة مع الطريقة الثانية، ومعامل التغاير لها اكبر بمقدار 0.5% فقط من الطريقة الثانية البديلة. لقد قورنت هاتان الطريقتان مع الطرق المعتمدة في اربع مدونات هي (ACI 318M-11، ACI 318M-99، BS، NZ).
بالاعتماد على نسبة مقاومة الانثناء العملية/مقاومة الانثناء المحسوبة M_{test}/M_r، أعطت الطريقتان المقترحتان قيما لمعامل التغاير قليلة نسبيا: 15.9% للطريقة المبسطة، 15.4% للطريقة البديلة. بينما كانت قيم معامل التغاير 17.5%، 15.9%، 15.6% و 15.7% للمدونات الاربع السابقة على التوالي.
هنالك فائدة أساسية عند تطبيق طريقة التصميم البسيطة (باستعمال قيمة ثابتة ϕ=0.85) وهي ان جميع النماذج (181 نموذجا) أعطت تقديرا أمينا.

Details of 181 beam tests [6-28]

Beam No.	Reference No.	f'_c (MPa)	ρ	b/d	c/d
1	6	21.1	0.008804	1.000000	0.170137
2	6	21.1	0.017608	1.000000	0.340274
3	6	32.8	0.008804	1.000000	0.129649
4	6	32.8	0.012276	1.000000	0.163093
5	6	40.5	0.008804	1.000000	0.119597
6	6	40.5	0.012276	1.000000	0.150448
7	7	21.7	0.035031	0.585586	0.468651
8	7	30.8	0.008294	0.576923	0.089643
9	7	38.4	0.018963	0.565217	0.186527
10	7	25.0	0.017912	0.572072	0.217703
11	7	15.2	0.022691	0.822581	0.433382
12	7	36.2	0.018912	0.556034	0.459984
13	7	25.9	0.008273	0.542017	0.098417
14	7	36.2	0.008403	0.533613	0.084463
15	7	10.3	0.017712	0.545852	0.504922
16	7	21.8	0.033821	0.547009	0.455539
17	7	8.6	0.013236	0.547414	0.446816
18	7	24.9	0.034277	0.560870	0.422427
19	7	32.3	0.034210	0.560870	0.361255
20	7	21.4	0.019245	0.560870	0.264061
21	8	12.3	0.011516	0.809524	0.261420
22	8	10.8	0.015497	0.822581	0.435626
23	8	13.3	0.022876	0.772727	0.583175
24	8	13.6	0.030361	0.822581	0.684397
25	8	20.5	0.010907	0.776119	0.148561
26	8	19.0	0.015012	0.796875	0.239881
27	8	19.1	0.023031	0.809524	0.394151
28	8	19.0	0.023719	0.822581	0.376242
29	8	19.1	0.030794	0.793651	0.552000
30	8	19.0	0.030003	0.868852	0.491668
31	8	19.1	0.039392	0.838710	0.642147
32	8	31.0	0.011161	0.784615	0.122082
33	8	31.0	0.014781	0.784615	0.166538
34	8	27.2	0.022978	0.796875	0.304129
35	8	31.0	0.023263	0.838710	0.241056
36	8	27.2	0.030190	0.809524	0.403579
37	8	31.0	0.031501	0.836066	0.354918
38	8	35.7	0.011701	0.822581	0.120068
39	8	35.7	0.015813	0.822581	0.167134
40	8	35.7	0.023087	0.822581	0.267171
41	8	35.3	0.024510	0.850000	0.239350
42	8	34.2	0.029105	0.796875	0.346348
43	8	35.7	0.031501	0.836066	0.313511
44	8	30.7	0.037587	0.787879	0.444205
45	8	35.7	0.045906	0.838710	0.531245
46	8	12.3	0.005189	0.825397	0.187922
47	8	14.3	0.009615	0.812500	0.441813
48	8	19.0	0.005376	0.822581	0.135134

Beam No.	Reference No.	f'_c (MPa)	ρ	b/d	c/d
49	8	22.1	0.009365	0.761194	0.270858
50	8	31.0	0.005376	0.822581	0.097494
51	8	28.6	0.009050	0.764706	0.219322
52	8	28.6	0.015152	0.772727	0.365248
53	8	30.7	0.009227	0.750000	0.226100
54	8	30.7	0.014277	0.787879	0.358107
55	8	32.9	0.023226	0.806452	0.415793
56	9	37.2	0.012035	0.720379	0.135262
57	9	36.2	0.018739	0.720379	0.239391
58	9	36.5	0.036169	0.720379	0.424616
59	9	15.4	0.021801	0.671053	0.583510
60	10	27.4	0.019100	0.541333	0.255942
61	10	28.2	0.019894	0.535809	0.214995
62	10	26.9	0.012797	0.544236	0.200585
63	10	32.6	0.012603	0.546917	0.158307
64	10	31.3	0.007906	0.548387	0.107097
65	10	29.8	0.007903	0.545455	0.125022
66	10	23.0	0.004404	0.545455	0.066124
67	10	25.0	0.004506	0.549865	0.077963
68	10	23.3	0.017810	1.201183	0.299721
69	10	28.7	0.017697	1.209581	0.282607
70	10	20.7	0.011106	0.749077	0.194404
71	10	27.6	0.010899	0.743590	0.168330
72	10	17.8	0.006204	0.424686	0.136326
73	10	23.3	0.006303	0.428571	0.112178
74	11	31.0	0.021474	0.769231	0.509683
75	11	29.5	0.021308	0.759494	0.512665
76	11	38.0	0.021807	0.779221	0.476576
77	12	27.2	0.011967	0.687783	0.236534
78	13	35.9	0.006528	0.584615	0.104026
79	13	37.3	0.014446	0.646809	0.226847
80	14	35.9	0.017407	0.740741	0.357108
81	14	39.4	0.017296	0.740741	0.343182
82	14	37.5	0.017500	0.740741	0.353114
83	14	39.3	0.017407	0.740741	0.345667
84	14	39.0	0.017500	0.740741	0.348370
85	14	39.9	0.017407	0.740741	0.344028
86	14	39.4	0.017407	0.740741	0.345387
87	15	27.6	0.014311	0.500000	0.259228
88	15	27.5	0.014311	0.500000	0.259763
89	15	27.4	0.014311	0.500000	0.260302
90	15	27.2	0.014311	0.500000	0.261396
91	15	26.7	0.014311	0.500000	0.264220
92	15	26.4	0.014311	0.500000	0.265977
93	15	25.7	0.014311	0.500000	0.270271
94	15	21.0	0.014311	0.500000	0.312613
95	15	27.1	0.007111	0.500000	0.130162
96	15	26.9	0.007111	0.500000	0.130720
97	15	26.4	0.007111	0.500000	0.132162
98	15	26.4	0.007111	0.500000	0.132162

Beam No.	Reference No.	f'_c MPa)	ρ	b/d	c/d
99	15	26.1	0.007111	0.500000	0.133060
100	15	25.9	0.007111	0.500000	0.133672
101	15	25.5	0.007111	0.500000	0.134932
102	15	25.1	0.007111	0.500000	0.136239
103	15	24.7	0.007111	0.500000	0.137597
104	15	24.4	0.007111	0.500000	0.138649
105	15	21.0	0.007111	0.500000	0.155336
106	15	27.3	0.005711	0.500000	0.104096
107	15	27.2	0.005711	0.500000	0.104315
108	15	27.0	0.005711	0.500000	0.104759
109	15	27.0	0.005711	0.500000	0.104759
110	15	26.8	0.005711	0.500000	0.105212
111	15	26.6	0.005711	0.500000	0.105673
112	15	26.4	0.005711	0.500000	0.106143
113	15	26.2	0.005711	0.500000	0.106621
114	15	26.1	0.005711	0.500000	0.106864
115	15	25.8	0.005711	0.500000	0.107605
116	15	25.5	0.005711	0.500000	0.108367
117	15	25.2	0.005711	0.500000	0.109151
118	16	21.0	0.014505	1.342105	0.316853
119	16	41.0	0.014505	1.342105	0.220896
120	17	42.0	0.016318	1.490196	0.246903
121	17	42.0	0.016318	1.490196	0.246903
122	17	42.0	0.016318	1.490196	0.246903
123	17	42.0	0.016318	1.490196	0.246903
124	17	21.0	0.016318	1.490196	0.356460
125	17	21.0	0.016318	1.490196	0.356460
126	17	21.0	0.016318	1.490196	0.356460
127	18	32.7	0.013076	1.200787	0.216936
128	18	32.7	0.013076	1.200787	0.216936
129	19	45.8	0.009181	1.315789	0.171337
130	19	36.3	0.013333	1.327434	0.301466
131	19	46.5	0.010572	1.515152	0.196843
132	19	46.2	0.015374	1.530612	0.325087
133	20	21.0	0.030000	0.716418	0.609216
134	20	110.0	0.030000	0.716418	0.220606
135	21	30.0	0.018985	0.597015	0.396915
136	21	30.0	0.005866	0.597015	0.122632
137	22	85.0	0.068447	0.851064	0.591992
138	22	85.0	0.068447	0.851064	0.591992
139	22	85.0	0.068447	0.851064	0.591992
140	22	85.0	0.038511	0.851064	0.402726
141	23	40.0	0.019286	0.666667	0.310832
142	23	60.0	0.019286	0.666667	0.248571
143	23	75.0	0.025850	0.666667	0.275737
144	23	40.0	0.002093	0.833333	0.030636
145	23	40.0	0.003533	0.833333	0.054067
146	23	40.0	0.019286	0.666667	0.310832
147	23	60.0	0.019286	0.666667	0.248571
148	23	75.0	0.003533	0.833333	0.034590
149	23	75.0	0.003533	0.833333	0.034590
150	23	75.0	0.025850	0.666667	0.275737
151	23	40.0	0.013673	0.666667	0.211766
152	23	40.0	0.019286	0.666667	0.310832
153	23	40.0	0.025850	0.666667	0.431002
154	23	40.0	0.041538	0.717949	0.604404

Beam No.	Reference No.	f_c' (MPa)	ρ	b/d	c/d
155	23	60.0	0.019286	0.666667	0.248571
156	23	71.0	0.019286	0.666667	0.210060
157	23	71.0	0.025850	0.666667	0.291271
158	23	71.0	0.041538	0.717949	0.452438
159	23	75.0	0.055714	0.717949	0.578154
160	24	31.0	0.005183	0.917431	0.088455
161	25	41.2	0.019286	0.666667	0.308247
162	25	41.0	0.019286	0.666667	0.308665
163	25	41.2	0.025850	0.666667	0.427415
164	25	41.2	0.025850	0.666667	0.427415
165	25	41.0	0.041538	0.717949	0.603166
166	25	41.2	0.041538	0.717949	0.602934
167	25	71.2	0.025850	0.666667	0.290518
168	25	71.2	0.025850	0.666667	0.290518
169	25	71.2	0.041538	0.717949	0.451271
170	25	71.2	0.041538	0.717949	0.451271
171	26	35.0	0.008619	0.746269	0.140784
172	27	32.3	0.003379	0.833333	0.052429
173	27	32.3	0.004180	0.833333	0.062296
174	27	32.3	0.003971	0.833333	0.052978
175	28	36.75	0.002778	0.555556	0.051279
176	28	36.75	0.002778	0.555556	0.051279
177	28	36.75	0.002778	0.555556	0.051279
178	28	36.75	0.005611	0.555556	0.103584
179	28	36.75	0.005611	0.555556	0.103584
180	28	36.75	0.005611	0.555556	0.103584
181	28	36.75	0.005611	0.555556	0.103584

Ratio of (M_{test}/M_r) for 181 beam tests[6-28]

Beam No.	Reference No.	ACI-11[1]	ACI-99[3]	BS-97[4]	NZ-95[5]	Simple Method	Alternative Method
1	6	1.01042	1.01042	1.00051	1.01042	1.06986	0.99997
2	6	1.20214	1.09729	1.14996	1.09729	1.16183	1.07240
3	6	1.10917	1.10917	1.07981	1.10917	1.17442	1.10738
4	6	1.13290	1.13290	1.11145	1.13290	1.19954	1.13056
5	6	1.08226	1.08226	1.04784	1.08226	1.14592	1.08436
6	6	1.06665	1.06665	1.03899	1.06665	1.12940	1.06930
7	7	1.59196	1.16812	1.33584	1.16812	1.23683	1.23879
8	7	1.19078	1.19078	1.14979	1.19078	1.26082	1.18862
9	7	1.14149	1.14149	1.12165	1.14149	1.20863	1.14343
10	7	1.21106	1.21106	1.21290	1.21106	1.28230	1.19848
11	7	1.44681	1.11748	1.22130	1.11748	1.18322	1.15240
12	7	1.37937	1.13512	1.29478	1.13585	1.20949	1.23718
13	7	1.40044	1.40044	1.35735	1.40044	1.48282	1.39479
14	7	1.29869	1.29869	1.25055	1.29869	1.37509	1.29877
15	7	1.62112	1.17081	1.39669	1.17081	1.23968	1.27292
16	7	1.55918	1.16536	1.31284	1.16536	1.23391	1.22365
17	7	1.77046	1.34015	1.49114	1.34015	1.41898	1.39750
18	7	1.37965	1.11671	1.20083	1.11671	1.18240	1.14858
19	7	1.18245	1.13086	1.17082	1.13086	1.19739	1.12425
20	7	1.14142	1.14142	1.16401	1.14142	1.20857	1.12216
21	8	1.29585	1.29585	1.32032	1.29585	1.37208	1.27423
22	8	1.66930	1.28492	1.41557	1.28492	1.36050	1.32759
23	8	1.52295	1.09991	1.37184	1.09991	1.16461	1.19810

Beam No.	Reference No.	ACI-11[1]	ACI-99[3]	BS-97[4]	NZ-95[5]	Simple Method	Alternative Method
24	8	1.98160	1.43115	1.80621	1.43115	1.51534	1.48753
25	8	1.09045	1.09045	1.07307	1.09045	1.15459	1.08071
26	8	1.12617	1.12617	1.13931	1.12617	1.19241	1.10913
27	8	1.47650	1.21582	1.30188	1.21582	1.28734	1.20898
28	8	1.48466	1.26242	1.34185	1.26242	1.33668	1.23191
29	8	1.69211	1.22208	1.51953	1.22208	1.29397	1.33931
30	8	1.64027	1.18464	1.40135	1.18464	1.25433	1.27683
31	8	1.87108	1.35134	1.69953	1.35134	1.43083	1.43937
32	8	1.19635	1.19635	1.16364	1.19635	1.26672	1.19347
33	8	1.02060	1.02060	1.00327	1.02060	1.08064	1.01719
34	8	1.11535	1.11535	1.14394	1.11535	1.18096	1.10205
35	8	1.08823	1.08823	1.09039	1.08823	1.15224	1.08276
36	8	1.37199	1.17445	1.24653	1.17445	1.24353	1.19237
37	8	1.23971	1.19247	1.23507	1.19247	1.26262	1.18313
38	8	1.28594	1.28594	1.24764	1.28594	1.36159	1.28577
39	8	1.12694	1.12694	1.10462	1.12694	1.19323	1.12673
40	8	1.19162	1.19162	1.19601	1.19162	1.26171	1.19123
41	8	1.14684	1.14684	1.14363	1.14684	1.21431	1.14608
42	8	1.14811	1.14467	1.17601	1.14467	1.21200	1.14163
43	8	1.16453	1.16453	1.18290	1.16453	1.23303	1.16407
44	8	1.43775	1.17809	1.25825	1.17809	1.24739	1.25189
45	8	1.51203	1.12775	1.24179	1.12775	1.19408	1.29637
46	8	1.23269	1.23269	1.22704	1.23269	1.30520	1.21848
47	8	1.55543	1.23653	1.47086	1.23653	1.30927	1.22891
48	8	1.13302	1.13302	1.11078	1.13302	1.19967	1.12388
49	8	1.15562	1.15562	1.18104	1.15562	1.22360	1.13571
50	8	1.17644	1.17644	1.13787	1.17644	1.24564	1.17421
51	8	1.11096	1.11096	1.10927	1.11096	1.17631	1.10337
52	8	1.30939	1.21765	1.27536	1.21765	1.28927	1.20270
53	8	1.13508	1.13508	1.13314	1.13508	1.20185	1.12944
54	8	1.27339	1.21491	1.26027	1.21491	1.28638	1.20468
55	8	1.44743	1.25594	1.32138	1.25594	1.32982	1.30563
56	9	1.23614	1.23614	1.20235	1.23614	1.30886	1.23690
57	9	1.22103	1.22103	1.21656	1.22103	1.29285	1.22126
58	9	1.33278	1.16199	1.21630	1.16199	1.23034	1.22677
59	9	1.70973	1.23481	1.52568	1.23481	1.30744	1.30514
60	10	1.02327	1.02327	1.03378	1.02327	1.08346	1.01351
61	10	1.12869	1.12869	1.12606	1.12869	1.19508	1.12071
62	10	0.95376	0.95376	0.94886	0.95376	1.00986	0.94639
63	10	1.06845	1.06845	1.04717	1.06845	1.13130	1.06618
64	10	1.10723	1.10723	1.07313	1.10723	1.17236	1.10505
65	10	1.11082	1.11082	1.08180	1.11082	1.17616	1.10739
66	10	1.21717	1.21717	1.17138	1.21717	1.28877	1.21289
67	10	1.10320	1.10320	1.06422	1.10320	1.16810	1.09937
68	10	0.98470	0.98470	1.01456	0.98470	1.04262	0.96745
69	10	1.17481	1.17481	1.19438	1.17481	1.24392	1.16429
70	10	1.14413	1.14413	1.14111	1.14413	1.21143	1.13044
71	10	1.14367	1.14367	1.12734	1.14367	1.21095	1.13698
72	10	1.17919	1.17919	1.15643	1.17919	1.24856	1.16959
73	10	1.10373	1.10373	1.07467	1.10373	1.16866	1.09717
74	11	1.38518	1.07903	1.28023	1.07956	1.15206	1.15668
75	11	1.38519	1.07473	1.28311	1.07473	1.14495	1.14195
76	11	1.30489	1.05898	1.18966	1.05898	1.12127	1.17468
77	12	1.02501	1.02501	1.02990	1.02501	1.08531	1.01584

Beam No.	Reference No.	ACI-11[1]	ACI-99[3]	BS-97[4]	NZ-95[5]	Simple Method	Alternative Method
78	13	1.24753	1.24753	1.20622	1.24753	1.32092	1.24748
79	13	1.22189	1.22189	1.21258	1.22189	1.29377	1.22330
80	14	1.15831	1.14007	1.17155	1.14007	1.20714	1.13990
81	14	1.12825	1.12825	1.14873	1.12825	1.19462	1.13357
82	14	1.12645	1.12645	1.15326	1.12645	1.19271	1.12892
83	14	1.04158	1.04158	1.06131	1.04158	1.10285	1.04640
84	14	1.03826	1.03826	1.05914	1.03826	1.09933	1.04267
85	14	1.03906	1.03906	1.05735	1.03906	1.10019	1.04467
86	14	1.04116	1.04116	1.06064	1.04116	1.10240	1.04611
87	15	1.27416	1.27416	1.28819	1.27416	1.34911	1.26213
88	15	1.29726	1.29726	1.31191	1.29726	1.37357	1.28483
89	15	1.30813	1.30813	1.32328	1.30813	1.38508	1.29541
90	15	1.32005	1.32005	1.33610	1.32005	1.39770	1.30683
91	15	1.33856	1.33856	1.35683	1.33856	1.41729	1.32415
92	15	1.34676	1.34676	1.36640	1.34676	1.42599	1.33165
93	15	1.36426	1.36426	1.38725	1.36426	1.44451	1.34742
94	15	1.35136	1.31164	1.36052	1.31164	1.38879	1.28473
95	15	1.51805	1.51805	1.48222	1.51805	1.60734	1.51087
96	15	1.52390	1.52390	1.48830	1.52390	1.61354	1.51648
97	15	1.53157	1.53157	1.49673	1.53157	1.62166	1.52359
98	15	1.54025	1.54025	1.50521	1.54025	1.63085	1.53222
99	15	1.54987	1.54987	1.51520	1.54987	1.64104	1.54146
100	15	1.55342	1.55342	1.51907	1.55342	1.64480	1.54476
101	15	1.56057	1.56057	1.52689	1.56057	1.65237	1.55140
102	15	1.56809	1.56809	1.53511	1.56809	1.66033	1.55840
103	15	1.57569	1.57569	1.54344	1.57569	1.66837	1.56545
104	15	1.57986	1.57986	1.54822	1.57986	1.67279	1.56921
105	15	1.56856	1.56856	1.54655	1.56856	1.66083	1.55386
106	15	1.57057	1.57057	1.52353	1.57057	1.66296	1.56485
107	15	1.58079	1.58079	1.53359	1.58079	1.67378	1.57495
108	15	1.58839	1.58839	1.54125	1.58839	1.68183	1.58235
109	15	1.58839	1.58839	1.54125	1.58839	1.68183	1.58235
110	15	1.59601	1.59601	1.54894	1.59601	1.68990	1.58977
111	15	1.60365	1.60365	1.55665	1.60365	1.69798	1.59720
112	15	1.61130	1.61130	1.56439	1.61130	1.70608	1.60464
113	15	1.61824	1.61824	1.57144	1.61824	1.71343	1.61136
114	15	1.61527	1.61527	1.56872	1.61527	1.71029	1.60832
115	15	1.61964	1.61964	1.57345	1.61964	1.71492	1.61239
116	15	1.62773	1.62773	1.58180	1.62773	1.72347	1.62015
117	15	1.63253	1.63253	1.58698	1.63253	1.72856	1.62463
118	16	1.18729	1.14109	1.18545	1.14109	1.20821	1.11730
119	16	1.22192	1.22192	1.20718	1.22192	1.29379	1.22699
120	17	1.42438	1.42438	1.41377	1.42438	1.50817	1.43233
121	17	1.16018	1.16018	1.15154	1.16018	1.22843	1.16666
122	17	1.38992	1.38992	1.37956	1.38992	1.47168	1.39768
123	17	1.37844	1.37844	1.36816	1.37844	1.45952	1.38613
124	17	1.67043	1.47517	1.55568	1.47517	1.56194	1.43979
125	17	1.48160	1.30841	1.37982	1.30841	1.38537	1.27703
126	17	1.48160	1.30841	1.37982	1.30841	1.38537	1.27703
127	18	1.57625	1.57625	1.56692	1.57625	1.66897	1.57167
128	18	1.57740	1.57740	1.56807	1.57740	1.67019	1.57282
129	19	1.48042	1.48042	1.44357	1.48042	1.56750	1.48915
130	19	1.25085	1.25085	1.26555	1.25085	1.32443	1.25131
131	19	1.73753	1.73753	1.70147	1.73753	1.83974	1.75019

Beam No.	Reference No.	ACI-11[1]	ACI-99[3]	BS-97[4]	NZ-95[5]	Simple Method	Alternative Method
132	19	1.26126	1.26126	1.26693	1.26126	1.33545	1.27689
133	20	2.32595	1.67985	2.07649	1.67985	1.77867	1.74872
134	20	1.20105	1.20105	1.17825	1.21379	1.27170	1.21379
135	21	1.19434	1.05444	1.11837	1.05444	1.11647	1.06923
136	21	0.97121	0.97121	0.94521	0.97121	1.02834	0.96837
137	22	1.35227	0.98183	1.02781	1.08341	1.03958	1.20061
138	22	1.33181	0.96697	1.01226	1.06701	1.02385	1.18244
139	22	1.36457	0.99076	1.03716	1.09326	1.04904	1.21153
140	22	1.11514	1.04460	1.06064	1.06646	1.10604	1.10012
141	23	1.26613	1.26613	1.27781	1.26613	1.34060	1.27235
142	23	1.29167	1.29167	1.27339	1.30729	1.36765	1.30729
143	23	1.24684	1.24684	1.23523	1.26375	1.32018	1.26375
144	23	1.79318	1.79318	1.70796	1.79318	1.89866	1.79395
145	23	1.51058	1.51058	1.44488	1.51058	1.59944	1.51174
146	23	1.25340	1.25340	1.26497	1.25340	1.32713	1.25957
147	23	1.30756	1.30756	1.28906	1.32338	1.38448	1.32338
148	23	1.60380	1.60380	1.52709	1.60628	1.69814	1.60628
149	23	2.31515	2.31515	2.20442	2.31873	2.45133	2.31873
150	23	1.20870	1.20870	1.19745	1.22510	1.27980	1.22510
151	23	1.12171	1.12171	1.10692	1.12171	1.18770	1.12531
152	23	1.20862	1.20862	1.21978	1.20862	1.27971	1.21456
153	23	1.35960	1.19614	1.24511	1.19614	1.26651	1.27863
154	23a	1.87927	1.34478	1.56786	1.35221	1.43709	1.61613
155	23	1.32720	1.32720	1.30842	1.34325	1.40527	1.34325
156	23	1.30626	1.30626	1.27912	1.31940	1.38310	1.31940
157	23	1.20017	1.20017	1.19239	1.21748	1.27077	1.21748
158	23	1.35516	1.16856	1.19900	1.19665	1.23730	1.29517
159	23	1.54124	1.13616	1.18417	1.25256	1.20299	1.37691
160	24	1.47305	1.47305	1.42187	1.47305	1.55970	1.47053
161	25	1.25973	1.25973	1.26864	1.25973	1.33383	1.26762
162	25	1.20425	1.20425	1.21319	1.20425	1.27508	1.21152
163	25	1.29995	1.15837	1.20167	1.15837	1.22651	1.23640
164	25	1.32535	1.18100	1.22514	1.18100	1.25047	1.26055
165	25	1.85080	1.32423	1.53479	1.33240	1.41532	1.60191
166	25	1.54960	1.10869	1.28346	1.11568	1.18499	1.34294
167	25	1.21035	1.21035	1.20234	1.22776	1.28155	1.22776
168	25	1.19957	1.19957	1.19163	1.21683	1.27013	1.21683
169	25	1.35100	1.16708	1.19718	1.19505	1.23573	1.29211
170	25	1.43020	1.23550	1.26736	1.26511	1.30818	1.36786
171	26	1.68241	1.68241	1.64014	1.68241	1.78137	1.68150
172	27	1.06000	1.06000	1.01494	1.06000	1.12236	1.05923
173	27	1.30851	1.30851	1.25548	1.30851	1.38548	1.30737
174	27	1.32941	1.32941	1.27304	1.32941	1.40762	1.32843
175	28	0.97757	0.97757	0.93508	0.97757	1.03508	0.97771
176	28	1.14180	1.14180	1.09217	1.14180	1.20897	1.14196
177	28	1.36860	1.36860	1.30911	1.36860	1.44911	1.36879
178	28	1.30694	1.30694	1.26315	1.30694	1.38382	1.30731
179	28	1.38615	1.38615	1.33971	1.38615	1.46768	1.38655
180	28	1.38615	1.38615	1.33971	1.38615	1.46768	1.38655
181	28	1.38615	1.38615	1.33971	1.38615	1.46768	1.38655